

Lesson: Volume of Solids: Rectangular Prisms. Cylinders. Spheres. Cones.

Finding Volume

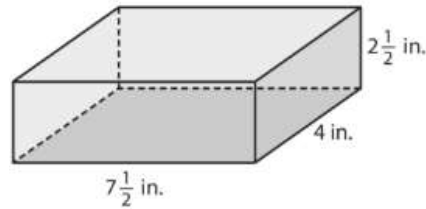
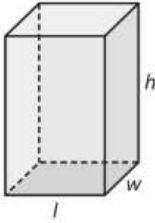
A rectangular prism has six faces. Any pair of opposite faces can be called the **bases** of the prism.

Example #1:

Find the volume of the rectangular solid.

Volume of a Rectangular Prism

$V = \ell wh$, or $V = Bh$
(where B represents the area of the prism's base; $B = \ell w$)



Example #2:

A terrarium is shaped like a rectangular prism. The prism is $25 \frac{1}{2}$ inches long, $13 \frac{1}{2}$ inches wide, and 16 inches deep. What is the volume of the terrarium?

Example #3:

The Smith family is moving and needs to decide on the size of the moving truck they should rent.

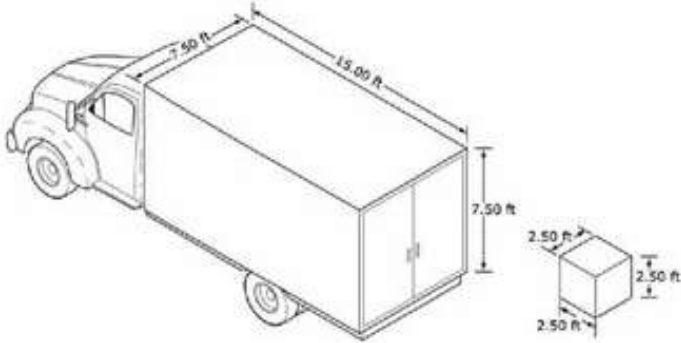
- a. A moving van rents for \$94.50 per day, and a small truck rents for \$162 per day. Based on the amount of space inside the van or truck, which is the better deal? Explain your answer.

- b. How much greater is the volume of the large truck than the volume of the small truck?

- c. The family estimates that they need about 1,100 cubic feet to move their belongings. What should they rent?

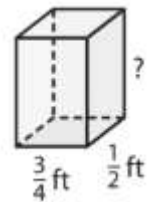
Inside Dimensions of Trucks			
Type	Length (ft)	Width (ft)	Height (ft)
Van	$10 \frac{1}{2}$	6	6
Small Truck	12	8	$6 \frac{3}{4}$
Large Truck	20	$8 \frac{3}{4}$	$8 \frac{1}{2}$

Example #4: How many boxes can fit inside the delivery truck shown below?



Example #5:

Find the height of this rectangular prism, which has a volume of $\frac{15}{16}$ cubic feet.



Example #6:

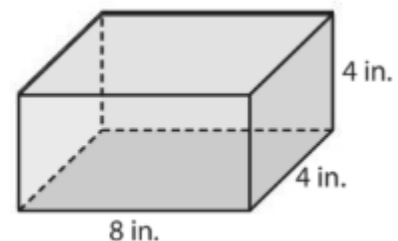
The classroom aquarium holds 30 gallons of water. It is 0.8 feet wide and has a height of 2 feet. Find the length of the aquarium.

Example #7:

History A typical stone on the lowest level of the Great Pyramid in Egypt was a rectangular prism 5 feet long by 5 feet high by 6 feet deep and weighed 15 tons. What was the volume of the average stone? How much did one cubic foot of this stone weigh?

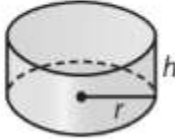
Example #8:

Hank has cards that are 8 inches by 4 inches. A stack of these cards fits inside the box shown and uses up 32 cubic inches of volume. How tall is the stack of cards? What percent of the box's volume is taken up by the cards?

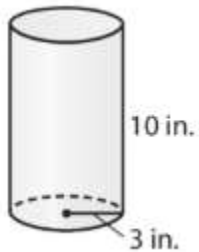


Finding volumes of cylinders is similar to finding volumes of prisms. You find the volume V of both a prism and a cylinder by multiplying the height h by the area of the base B , so $V = Bh$.

The base of a cylinder is a circle, so for a cylinder, $B = \pi r^2$.

Volume of a Cylinder	
The volume V of a cylinder with radius r is the area of the base B times the height h .	
$V = Bh$ or $V = \pi r^2 h$	

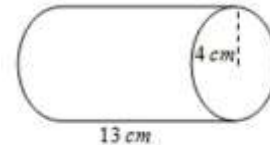
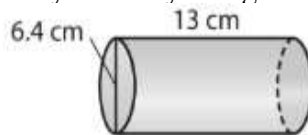
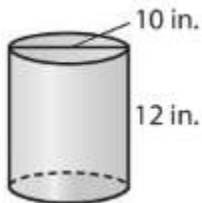
Example #1: Find the volume of the cylinder below.



a. in terms of π .

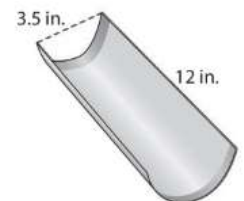
b. to the **nearest 10^{th}** . Use the π button on your calculator.

Example #2: Find the volume of each cylinder by using the approximation 3.14 for π .



Example #3:

A pan for baking French bread is shaped like half a cylinder. It is 12 inches long and 3.5 inches in diameter. What is the volume of uncooked dough that would fill this pan?



Example #4:

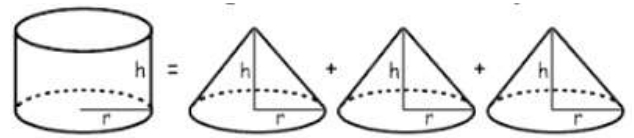
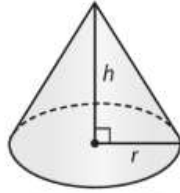
A circular swimming pool can hold 7850 cubic feet of water. The diameter of the pool is 50 feet. Find the height of the swimming pool. Use 3.14 for π .

Finding the Volume of a Cone Using a Formula

Volume of a Cone

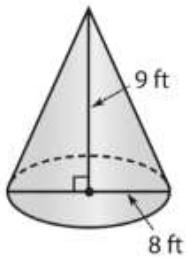
The volume V of a cone with radius r is one third the area of the base B times the height h .

$$V = \frac{1}{3} Bh \text{ or } V = \frac{1}{3} \pi r^2 h$$

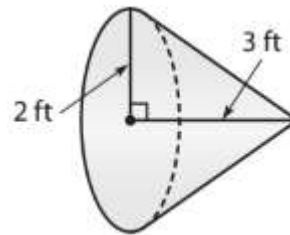


The volume of a cylinder is equal to the volume of 3 cones with same radius and height.

Example #1: Find the volume in terms of π .



Example #2: Find the volume to the *nearest 10th*.



Example #3:

For her geography project, Karen built a clay model of a volcano in the shape of a cone. Her model has a diameter of 12 inches and a height of 8 inches. Find the volume of clay in her model to the nearest tenth. Use 3.14 for π .

Example #4:

Find the missing measure for each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

radius = _____

diameter = 6 cm

height = 6 in.

height = _____

volume = 100.48 in³

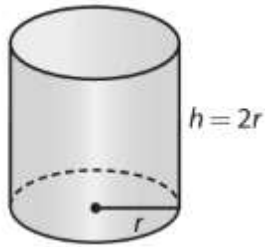
volume = 56.52 cm³

Example #5: Find the volume of the pencil shown. The pencil is a **compound shape** of a **cylinder** and a **cone**. Use $\pi \approx 3.14$ and round to the *nearest 10th*.



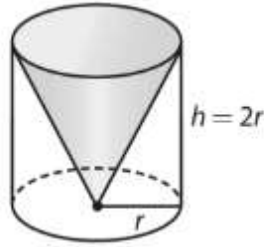
Finding the Volume of a Sphere Using a Formula

A sphere having a **diameter** that is **equal** to the **height** of a cylinder will **fill the cylinder 2/3 of the way**.



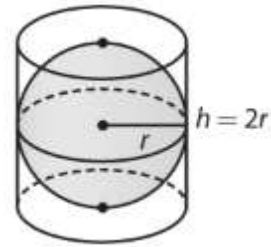
Cylinder

$$V = \pi r^2 h$$



Cone

$$V = \frac{1}{3} \pi r^2 h$$



Sphere

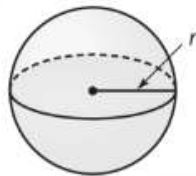
$$V = \frac{2}{3} \pi r^2 h$$

Replace h with $2r$.

Volume of a Sphere

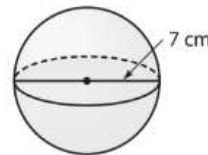
The volume V of a sphere is $\frac{4}{3}\pi$ times the cube of the radius r .

$$V = \frac{4}{3} \pi r^3$$

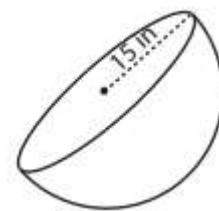
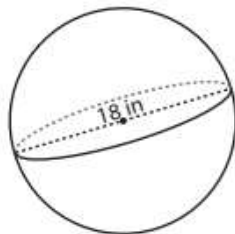
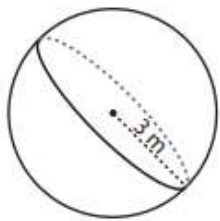


Example #1:

Find the volume in terms of π .



Example #2: Find the volume of each solid to the **nearest 10th**. Use $\pi \approx 3.14$. The last diagram is half of a sphere.



Example #3: What is the volume of a soccer ball with a diameter of 22 centimeters? Use $\pi \approx 3.14$.



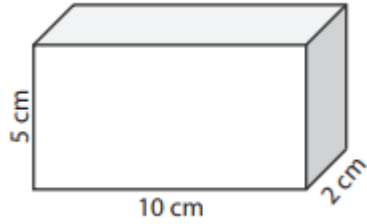
Example #4:

Multistep A cylindrical can of tennis balls holds a stack of three balls so that they touch the can at the top, bottom, and sides. The radius of each ball is 1.25 inches. Find the volume inside the can that is not taken up by the three tennis balls.

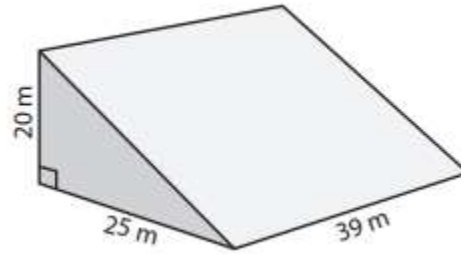


Mixed Volume Practice. Find the volume of each shape for #1-5.

1.

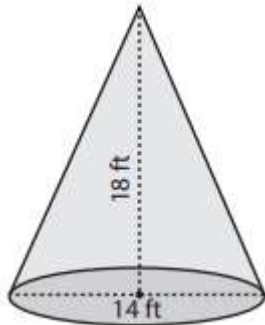


5.

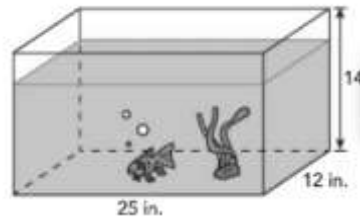


Think

2.

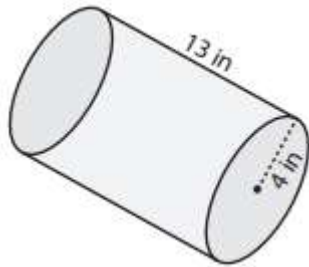


6.



How many cubic inches of water is needed to fill this tank $\frac{3}{4}$ of the way up?

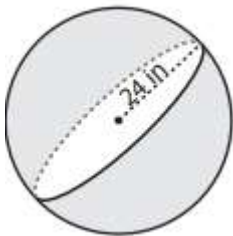
3.



7.

Andrew's job is to attach a ladder to a water tank. The water tank is a 20 feet wide cylinder and has a volume of 9,420 cubic feet. If the length of the ladder is equal to the height of the water tank, how long is the ladder?

4.



- a. 30 ft
- b. 9.5 ft
- c. 8.9 ft
- d. 11 ft

9.

Ace Canning Company is redesigning its product. The new cans will have twice the radius, but the same height of the original cans. Which statement is true about the volume of the new cans compared to the volume of the original cans?

- a. The volumes of the new and original cans will be equal.
- b. The volume of the new cans will be twice the volume of the original cans.
- c. The volume of the new cans will be four times the volume of the original cans.
- d. There is not enough information to determine.