Name:  Answers  
Date:  
Algebra 2 Common Core

Unit 9 Part II Review Sheet

1. Write the function with the following parameters:

<table>
<thead>
<tr>
<th>Function</th>
<th>Amplitude</th>
<th>Period</th>
<th>Horizontal</th>
<th>Vertical Shift</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosine</td>
<td>0.6</td>
<td>$\frac{2\pi}{\frac{1}{2}}$</td>
<td>None</td>
<td>None</td>
<td>$y = 0.6 \cos \left(\frac{1}{2}x\right)$</td>
</tr>
<tr>
<td>Sine</td>
<td>5</td>
<td>$\frac{2\pi}{3}$</td>
<td>None</td>
<td>Up 2</td>
<td>$y = 5 \sin (3x) + 2$</td>
</tr>
</tbody>
</table>

2. Write the equation of each curve as a sine and cosine function

![Graph of y = 4 cos (2x) + 1]

- **Mid** $y = 1$
- **Amp** = 4
- **FREQ** = $\frac{360^\circ}{\frac{180^\circ}{\frac{2\pi}{\pi}}} = 2$

Mid $\Rightarrow y = 1$

- **PER** = $180^\circ$ or $\pi$  

\[
\text{Cycle} = 180^\circ \text{ so } \text{PER} = 180^\circ \text{ or } \pi
\]

a)

![Graph of y = 2.4 sin (\frac{\pi}{2}x)]

- **Mid** = $y = 0$
- **PER** = 4
- **FREQ** = $\frac{2\pi}{4} = \frac{\pi}{2}$

Mid $\Rightarrow y = 0$

- **Amp** = 2.4

(1 cycle takes 4 units $\Rightarrow$ PER = 4)
3. The period of the function with rule \( f(x) = 3\cos(2\pi x) \) is:

\[
\text{Per} = 1
\]

\[
\begin{align*}
\text{FREQ} = \frac{2\pi}{\text{PER}} & = \frac{2\pi}{1} = 2\pi \\
\Rightarrow \text{Per} & = \frac{2\pi}{2\pi} = 1
\end{align*}
\]

4. The minimum value of \( y = 3 + 2\cos(x) \) is:

\[
\min = \text{midline} - \text{AMP} \Rightarrow 3 - 2 = 1
\]

5. An angle of \( \frac{2\pi}{7} \) radians expressed in degrees (correct to two decimal places) is:

\[
\frac{2(180)}{7} = 51.43^\circ
\]

6. The smallest value of \( x \) for which the graph of \( y = 2\cos(2x) + 2 \) touches the \( x \)-axis is:

\( x = \pi \)

7. The function \( f \) where \( f(x) = -2\sin(x) + 2 \) has range and amplitude respectively of:

\[
\begin{align*}
\text{MAX} & = \text{MID} + \text{AMP} \Rightarrow 2 + 2 = 4 \\
\text{MIN} & = \text{MID} - \text{AMP} \Rightarrow 2 - 2 = 0 \\
\text{Range} & = [0, 4] \text{ and } \text{AMP} = 2
\end{align*}
\]
8. The function \( f(x) = a \cos(bx) \), where \( a, b \) and \( c \) are positive constants, has period:

A. \( a \)
B. \( b \)
C. \( \frac{2\pi}{a} \)
D. \( \frac{2\pi}{b} \)
E. \( \frac{b}{2\pi} \)

\[ \text{Per} = \frac{2\pi}{\text{Freq}} \quad \therefore \quad \text{Per} = \frac{2\pi}{b} \]

9. The vertical distance from the ground of a point on a wheel as it rotates is given by \( D(t) = 4 - 4 \sin \left( \frac{8\pi}{t} \right) \) where \( t \) is the time in minutes. What is the time, in seconds, for a full rotation of the wheel?

A. 10
B. 15 √
C. 30
D. 60
E. 90

\[ \text{Per} = \frac{2\pi}{\text{Freq}} = \frac{2\pi}{8\pi} = \frac{1}{4} \text{ of a minute} \quad \text{so... 15 seconds!} \]

10. Show all work and round to the nearest tenth.

a)
\[
\begin{align*}
\cos 41^\circ &= \frac{x}{25} \\
x &= 25 \cos 41^\circ = 18.9
\end{align*}
\]

b)
\[
\begin{align*}
\tan x^\circ &= \frac{14}{8} \\
\tan^{-1} \left( \frac{14}{8} \right) &= 60.3^\circ
\end{align*}
\]

11. State the quadrant in which the terminal side of \( \theta \) lies if \( \tan \theta < 0 \) and \( \cos \theta > 0 \).

\[ \text{IV} \]
12. State one positive and one negative co-terminal angle for each of the following.

\[
\begin{align*}
a) \quad \frac{2\pi}{3} \quad & \quad \frac{2(180)}{3} = 120^\circ \\
\end{align*}
\]

\[
\begin{align*}
b) \quad -\frac{3\pi}{4} \quad & \quad -3(180) = -135^\circ \\
\end{align*}
\]

13. During its approach to Earth, the space shuttle’s glide angle changes. When the shuttle’s altitude is about 15.7 miles, its horizontal distance to the runway is about 59 miles. What is its glide angle? Round your answer to the nearest tenth.

\[
\tan X = \frac{15.7}{59}
\]

\[
\tan^{-1}(\frac{15.7}{59}) = 14.9^\circ
\]

14. Evaluate and find the exact value:

\[
\begin{align*}
a) \quad \sin(-\frac{\pi}{6}) = \sin(-30^\circ) = -\frac{1}{2} \\
b) \quad \csc\frac{7\pi}{3} = \csc(420^\circ) \\
& \quad \sin 420^\circ = \frac{\sqrt{3}}{2} \quad \text{so FLIP!} \\
& \quad \csc(420^\circ) = \frac{2}{\sqrt{3}} \\
c) \quad (\tan\frac{11\pi}{6})(\sin\frac{7\pi}{4}) \\
& \quad (-\frac{\sqrt{3}}{3})(-\frac{\sqrt{2}}{2}) = \frac{\sqrt{6}}{6}
\end{align*}
\]

15. Convert the following from degrees to radians or vice versa and state the quadrant it terminates in:

\[
\begin{align*}
a) \quad 36^\circ \quad & \quad \frac{360\text{ }^\circ}{180\text{ }^\circ} = \frac{2\pi}{5} \quad \text{I} \\
\end{align*}
\]

\[
\begin{align*}
b) \quad 17\frac{\pi}{20} \quad & \quad 17\frac{180}{2} = 153^\circ \quad \text{II} \\
\end{align*}
\]

\[
\begin{align*}
c) \quad 195^\circ \quad & \quad \frac{195\pi}{180} = \frac{9\pi}{4} \quad \text{III}
\end{align*}
\]

16. Find the values of the six trigonometric functions of \(\theta\), if \(\theta\) is an angle in standard position with the point \((-5, -12)\) on its terminal ray.

\[
\begin{align*}
\sin \theta = \frac{-12}{13} \\
\cos \theta = \frac{-5}{13} \\
\tan \theta = \frac{-12}{5} \\
\csc \theta = \frac{-13}{12} \\
\sec \theta = \frac{-13}{5} \\
\cot \theta = \frac{5}{12}
\end{align*}
\]
17. Graph each of the following on the grid provided if \(0 \leq x < 2\pi\) and state the range for each.

\[ y = 2\sin(x) + 2 \]

\[ \text{AMP: 2} \]
\[ \text{MIDLINE: } y = 2 \]
\[ \text{PER: } \frac{2\pi}{1} = 2\pi \]
\[ \text{FREQ: 1} \]

18. The elk population in Denali National Park dropped from a high of 150,000 in 1950 to a low of 60,000 in 1975. However the population then rose back to a maximum of 150,000 again in 2000. Scientists hypothesize that this pattern will continue and that the elk population will fluctuate in this manner every 50 years.

a) Letting \(t = 0\) in 1950, sketch the graph of this elk population from 1950 through 2000.

\[ y = 45,000 \cos \left( \frac{\pi}{25} x \right) + 105,000 \]

b) List the following information about the equation above:

\[ \text{AMP} = 45,000 \]
\[ \text{MIDLINE} = 105,000 \]
\[ \text{FREQ} = \frac{2\pi}{50} = \frac{\pi}{25} \]
\[ \text{PERIOD} = 50 \text{ yrs} \]

\[ + \cos \]

c) Using your equation, what would you estimate the elk population to be in 2020? (Round your answer to the nearest whole elk!)

\[ y = 45,000 \cos \left( \frac{\pi}{25} (70) \right) + 105,000 \]

or:

\[ y = 45,000 \cos \left( \frac{70}{25} \right) + 105,000 \]

\[ \text{plug in } x = 70 \]

\[ 68,594.235 \approx 68,594 \]