UNIT 8 REVIEW SHEET #2

Part I Questions

1. A sequence is defined by the rule \( b_k = (b_{k-1})^2 - k \) with \( b_1 = 3 \). Which of the following represents the value of \( b_3 \)?

(1) 28  
(2) 40

\[ \begin{align*}
(\text{k=1}) & \quad b_1 = 3 \\
(\text{k=2}) & \quad b_2 = (3^2) - 2 = 7 \\
(\text{k=3}) & \quad b_3 = (7^2) - 3 = 46
\end{align*} \]

2. If 24, x, and 6 form the first three terms of an arithmetic sequence, then which of the following is the value of x?

(1) 12  
(2) 15

\[ \begin{align*}
2y + 2d &= 6 \\
-2y &= -2y \\
2d &= 16 \\
2 &= 2
\end{align*} \]

3. If \( f(n) = f(n-1) + 5 \) and \( f(1) = -2 \) then \( f(20) = ? \)

(1) 93  
(2) 98

\[ \begin{align*}
f(1) &= -2 \\
f(2) &= -2 + 5 = 3 \\
f(3) &= 3 + 5 = 8 \\
f(4) &= 8 + 5 = 13 \\
\end{align*} \]

4. The function \( g \) is defined by \( g(x) = \sum_{i=3}^{5} \left( \frac{x}{i} + i \right) \). Which of the following is the value of \( g(60) \)?

(1) 47  
(2) 50

\[ \sum_{i=3}^{5} \left( \frac{60}{i} + i \right) = 59 \]
5. If the pattern shown below in the stacks of dodecahedrons continues, how many will be in the 20th stack?

(1) more than 1,000 but less than 5,000
(2) more than 100,000 but less than 1 million
(3) between one and two million
(4) more than 5 million

\[ a_1 = 2 \quad r = 2 \]

\[ a_{20} = 2 \cdot (2)^{20-1} = 104,857,600 \]

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6. Gemma plans to run 5 miles her first week and increase the amount she runs each week by 20%. Which of the following is closest to the total distance Gemma has run after 10 weeks?

(1) 115 miles
(2) 130 miles
(3) 138 miles
(4) 145 miles

\[ \sum_{n=1}^{10} 5(1.2)^{n-1} = 129,793.4 \]

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7. Which of the following represents the sum

\[ \frac{4}{14} + \frac{5}{23} + \frac{6}{34} + \frac{7}{47} + \frac{8}{62} \]?

(1) \[ \sum_{n=1}^{5} \frac{n+3}{n+5} \]
(2) \[ \sum_{n=1}^{8} \frac{n}{n^2 - 2} \]
(3) \[ \sum_{n=1}^{7} \frac{n+3}{n+2} \]
(4) \[ \sum_{n=3}^{8} \frac{n}{n+2} \]

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X SKIP!

If a mortgage of $230,000 has a yearly interest rate of 4.5% applied monthly and is being paid off using monthly payments of $1,400, how much of the first payment went towards reducing the amount owed on the mortgage?

(1) $537.50
(2) $652.70
(3) $862.50
(4) $1,043.80
II QUESTIONS:

9. A sequence is defined by the rule \( a_i = i \cdot a_{i-1} \) with \( a_1 = 4 \). Determine the value of \( a_4 - a_2 \).

\[
\begin{align*}
q_4 - q_3 &= \sqrt{72} \\
a_1 &= 4 \\
a_2 &= \frac{2}{3} \cdot 4 = 8 \\
a_3 &= \frac{3}{2} \cdot 8 = 24 \\
a_4 &= \frac{4}{3} \cdot 24 = 96 \\
96 - 8 &= 88
\end{align*}
\]

10. A geometric sequence is given below. Find a non-recursive formula for the \( n^{th} \) term of this sequence.

18, \underline{18}, 18, \underline{5}, 25, 125, ...

\( a_1 = 18 \), \( r = \frac{5}{18} \)

\[ a_n = 18 \left( \frac{5}{18} \right)^{n-1} \]

11. Express the following sum using summation (also known as sigma) notation.

\[ \sum_{n=1}^{8} (\frac{n}{n+2}) \]

*PART III QUESTIONS:

12. If the expressions \( x + 5, 2x, \) and \( 4x - 17 \) represent the first three terms of an arithmetic sequence, then determine the numerical value of the 20\(^{th} \) term.

\( \begin{align*}
\mathcal{Q}_1 &= 2x - (x+5) = x - 5 \\
\mathcal{Q}_2 &= 4x - 17 - (2x) = 2x - 17 \\
d_1 &= d_2 \implies 2x - 17 = x - 5 \\
x &= 12
\end{align*} \)

\( \begin{align*}
\mathcal{Q}_1 &= x + 5 = 17 \\
\mathcal{Q}_2 &= 2x = 24 \\
\mathcal{Q}_3 &= 4x - 17 = 31 \\
\mathcal{Q}_{20} &= 17 + (20 - 1)(7) = 150
\end{align*} \)
13. The following series is arithmetic:

\[ S_n = 8 + 13 + 18 + \ldots + 138 \]

(a) Determine the number of terms in this series. Show how you arrived at your answer.

\[
\begin{align*}
    a_n &= a_1 + (n-1)d \\
    138 &= 8 + (n-1)(5) \\
    138 - 8 &= (n-1)(5) \\
    130 &= 5n - 5 \\
    135 &= 5n \\
    n &= 27
\end{align*}
\]

(b) Determine the sum of this series. A numerical answer alone will earn at most one point.

\[
\sum_{n=1}^{27} \left( 8 + (n-1)(5) \right) = 1971
\]

**PART IV QUESTION:**

14. A car loan of $24,000 is being paid off with monthly payments of $250. The balance on the loan is being charged an annual interest of 4% applied on a monthly basis: \((1 + \frac{r}{12}) = (1 + \frac{0.04}{12}) = (1.00333)\)

(a) Determine how much is owed on the loan after two months of payments. Show how you arrived at your answer.

\[
\begin{align*}
\text{START WITH: $24,000} & \\
\text{AFTER 1 MONTH!} & \quad 24,000 \times (1.00333) - 250 = 23,829.999 \\
\text{AFTER 2 MONTHS!} & \quad 23,829.999 \times (1.00333) - 250 = 23,659.433
\end{align*}
\]

(b) Write a recursively defined rule that calculates the balance, \(b_n\), owed after \(n\)-payments based on the balance owed, \(b_{n-1}\). You do not need to give an initial value.

\[
b_n = (b_{n-1})(1.00333) - 250
\]

(c) Explain why the sequence of balances owed is *not* geometric?

*It does not have a common ratio!"