REVIEW OF UNIT 6 PART 1: EXPONENTIAL FUNCTIONS

Before we get started let's fill in some facts:

- Formula for Annual Growth: \[ y = a (1 + r)^t \]
- Formula for Annual Decay: \[ y = a (1 - r)^t \]
- Formula for Continuous Growth/Decay: \[ y = a e^{rt} \]
- Formula for Compound Interest: \[ y = a (1 + \frac{r}{n})^{nt} \]
- Domain of an Exponential in the form \( y = b^x \): \[ \text{All reals} \]
- Range of an Exponential in the form \( y = b^x \): \[ y > 0 \]
- Equation for the Horizontal asymptote for an Exponential in the form \( y = b^x \): \( x = A_x(\text{At}) \)

Matching \( \left( 5 \right) \)

Match the situation below with its exponential model.

- a. \( y = 34(1.122)^t \)
- b. \( y = 8851(0.883)^t \)
- c. \( y = 8851(1.025)^t \) \( \text{in millions} \)
- d. \( y = 34(2)^t \)
- e. \( y = 8851(0.975)^t \)
- f. \( y = 8851(1.07)^t \)
- g. \( y = 8851(1.117)^t \)
- h. \( y = 34(0.891)^t \)
- i. \( y = 8851(1.068)^t \)
- j. \( y = 34(0.9)^t \)

1. The population of Mexico City, Mexico (in thousands) after \( t \) years when its current population is 8,851,000 with 2.5% growth each year.
2. The value of a boat \( t \) years after it was purchased for \$8851 when it decreases in value by 11.7% each year.
3. The number of amoeba present \( t \) days after you started an experiment with a sample of 34 amoeba that doubles every 6 days.
4. The amount (in grams) of the radioactive isotope manganese-52 remaining after \( t \) days when the initial amount is 34 grams and halves every 6 days.
5. The amount (in dollars) in an account after \( t \) years that pays 6.8% annual interest when the initial principal is \$8851 and the interest is compounded monthly.
1. If $5000 is invested at a rate of 3% compounded quarterly, what is the value of the investment in 5 years?

(1) $5190.33  
(2) $5796.37  
(3) $5805.92  
(4) $5808.08

\[ y = 5000 \left(1 + \frac{0.03}{4}\right)^{4 \times 5} = 5805.92 \]

2. What is the domain of \( f(x) = 2^x + 2 \)?

(1) All integers  
(2) All real numbers  
(3) \( y > 0 \)  
(4) \( y > 2 \)

3. The value of \( x \) in the equation \( 4^{2x+5} = 8^{3x} \) is

(1) 1  
(2) 2  
(3) 5  
(4) -10

\[ 2(2x+5) = 3(3x) \]

4. Akeem invests $25,000 in an account that pays 4.75% annual interest compounded continuously. Using the formula \( A = Pe^{rt} \), where \( A \) is the amount in the account after \( t \) years, \( P \) is principal invested, and \( r \) is the annual interest rate, how many years, to the nearest tenth, will it take for Akeem's investment to double?

(1) 10.0  
(2) 14.6  
(3) 23.1  
(4) 24.0

\[ 25000 e^{0.0475 \times t} = 50000 \]

5. Which equation is represented by the graph below?

(1) \( y = x \)  
(2) \( y = 0.5^x \)  
(3) \( y = 5^x \)  
(4) \( y = 8.5^x \)

6. Which statement about the graph of the equation \( y = e^x \) is not true?

(1) It is asymptotic to the \( x \)-axis  
(2) The domain is the set of all real numbers  
(3) It lies in Quadrants I and II  
(4) It passes through the point \((0, 2)\)
For each equation below, solve algebraically for \( x \): 

7. \( 27^{-3x} = 81^{1-3x} \)

\[
\begin{align*}
3(-3x) &= 4(1-3x) \\
-9x &= 4 - 12x \\
-3x &= 4 \\
x &= \frac{4}{3}
\end{align*}
\]

8. \( 4^{5x} = \left( \frac{1}{64} \right)^{x+1} \)

\[
\begin{align*}
5x &= \left( \frac{1}{4} \right)^{x+1} \\
5x &= \left( \frac{1}{4} \right)^x \\
5x &= -3(x+1) \\
5x &= -3x - 3 \\
x &= -\frac{3}{8}
\end{align*}
\]

9. \( \frac{4 \cdot 2^{-x}}{2^{x-4}} = 16 \)

\[
\begin{align*}
2^{-x} &= 64 \\
2^{-x} &= 2^6 \\
x &= -6
\end{align*}
\]

10. The current population of Little Pond, New York, is 20,000. The population is \textit{decreasing}, as represented by the formula \( P = A(1.3)^{-0.234t} \) where \( P \) = final population, \( t \) = time, in years, and \( A \) = initial population.

What will the population be 5 years from now? Round your answer to the \textit{nearest hundred people}.

\[
P = 20000 \cdot (1.3)^{-0.234(5)} = 14,713.50948 \\
\sqrt{14,713.50948} \approx 383 \text{ people}
\]

11. Kathy deposits \$25,000 into an investment account with an annual rate of 5\%, \textit{compounded continuously}.

a) Write a formula to represent the value in Kathy’s account in \( t \) years.

\[
y = a e^{rt} \]

b) How much money will be in the account after 12 years? Round to the \textit{nearest hundred dollars}.

\[
y = 25000 e^{0.05(12)} = 84552.97 \approx 84550 \text{ dollars}
\]

12. The Smith family needs to have \$35,000 available in 18 years when their son is ready for college. If they have decided to invest in a mutual fund that \textit{compounds interest daily} at 5.2\%, how much money do they need to invest now to reach their goal?

\[
y = \frac{35000}{ \left( 1 + \frac{.052}{365} \right)^{365(18)}} \\
an = 365 \\
\rightarrow r = .052 \\
n = 365 \\
t = 18 \\
\]

\[
y = \frac{35000}{ \left( 2.54959164 \right)^{18}} \\
\rightarrow y = 13,727.69 \text{ dollars}
\]
13. If I deposit $2,000 into an account with a nominal interest rate of 1.2% compounded monthly, what's the value of the account in 2 years?

\[ y = a \left(1 + \frac{r}{n}\right)^{nt} \]

\[ a = 2000 \]
\[ r = 0.012 \]
\[ n = 12 \]
\[ t = 2 \]

\[ y = 2000 \left(1 + \frac{0.012}{12}\right)^{12(2)} \]

\[ \approx 2048.56 \]

14. Given the equation \( P(t) = 450(0.85)^t \), where \( P \) is the population of Lonely Town, \( t \) years after 2010. Answer the following questions:

(a) What was the population in 2010?

\[ \text{INITIAL!} \]

\[ 450 \]

(b) Is the population increasing or decreasing? Explain. At what rate?

\[ 1 + r \text{ is } > (\text{greater than}) \]
\[ 1 - r \text{ is } < (\text{less than}) \]

Decreasing by 15% since \( 1 - r = 0.85 \)

\[ \frac{-0.15}{-1} \]

\[ r = 0.15 = 15\% \]

15. George invests $1200 money into an account with an interest rate of 4.75% compounded continuously. Brad invested in Account Z and his account is shown below.

Who has a larger principal? Who will have more money in 10 years?

\[ \text{INITIAL } \Rightarrow \text{ BRAD: had 1000 initially} \]
\[ \text{GEORGE: had 1200 initially} \]

In 10 years: BRAD had approximately $1700 (from graph).

\[ \text{GEORGE has } y = 1200 e^{0.0475(10)} = 1929.617 \]

\[ \text{GEORGE has the greater principal} \]

\[ \text{GEORGE will have more} \]