Unit 3 & 4 Review Questions

1. Factor each of the following:
   a) \( 2x^2 - 2x - 12 \)
   \[ \frac{2(x^2 - x - 6)}{2(x - 3)(x + 2)} \]
   b) \( x^3 - 4x \)
   \[ \frac{x(x^2 - 4)}{x(x + 2)(x - 2)} \]
   c) \( x^3 + 2x^2 + 2x + 4 \)
   \[ \frac{x^2(x + 2) + 2(x + 2)}{(x + 2)(x^2 + 2)} \]

2. Solve for \( x \) by factoring:
   \( 4x^2 - 20x - 5 = -2x^2 + 9x \)
   \[ 6x^2 - 29x - 5 = 0 \]
   \[ 6x^2 - 30x - 11 = 0 \]
   \( x = \frac{-5}{6}, x = 1 \)

3. Solve for \( x \) using completing the square and express the roots in simplest \( a + bi \) form:
   \( 3x^2 - 12x = -21 \)
   \[ \frac{3x^2 - 12x}{3} = -7 \]
   \[ x^2 - 4x = -7 \]
   \[ \frac{(x - 2)^2}{4} = -3 \]
   \[ x - 2 = \pm \sqrt{3} \]
   \[ x = 2 \pm i\sqrt{3} \]

4. Solve for \( x \) by using the quadratic formula and express the roots in simplest \( a + bi \) form:
   \( 3x^2 - 6x = -4 \)
   \[ \frac{3x^2 - 6x}{3} < 0 \]
   \[ a = 3, b = -6, c = 4 \]
   \[ x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \]
   \[ x = \frac{-6 \pm \sqrt{36 - 48}}{6} \]
   \[ x = \frac{-6 \pm 2i\sqrt{3}}{6} \]
   \[ x = \frac{1}{3} \pm i\sqrt{3} \]

5. Perform the indicated operations and express your answer in simplest \( a + bi \) form.
   a) \((6 + \sqrt{-49}) + (3 + \sqrt{-64})\)
   \[ (6 + 7i) + (3 + 8i) \]
   \[ 9 + 15i \]
   b) \(-(-1 + 2\sqrt{-12}) - (8 + 5\sqrt{-48})\)
   \[ (-1 + 2(2i\sqrt{3})) - (8 + 5(4i\sqrt{3})) \]
   \[ -9 - 16i\sqrt{3} \]

6. Perform the following complex calculation. Express your answer in simplest \( a + bi \) form.
   \[ 7(3x - 5i) + (4x - 2i)(-6x + 7i) \]
   \[ i^2 = -1 \]
   \[ 21x - 35i + [-24x^2 + 28xi - 12xi - 14i^2] \]
   \[ 21x - 35i + (-24x^2 + 40xi + 14i) \]
   \[ -24x^2 + 40xi + 31x + 14i \]
   (Terms can be in any order.)
Unit 4 Sample Questions

2. For the polynomial function graphed below, which of the following intervals given is the function strictly increasing?

(1) \( x > 0 \)  
(2) \( x < 1 \)  
(3) \( 0 < x < 2 \)  
(4) \( x > \)

3. What is the value of \( k \) such that \((2x^3 - kx^2 + x - 6) + (x - 2)\) has a remainder of zero?

(1) -2  
(2) 2  
(3) \( 3 \)  
(4) -6

Plug in \( x = 2 \) for \( 12 - 4k = 0 \)

\[ 2 (2)^3 - k (2)^2 + (2) - 6 = 0 \]
\[ 16 - 4k - 4 = 0 \]
\[ 12 = 4k \]
\[ k = 3 \]

4. The function \( f(x) \) is an odd function with \( f(3) = 7 \) and \( f(-9) = 11 \). Then \( f(-3) \) must be equal to

(1) 7  
(2) -7  
(3) 3  
(4) -11

\[ \text{(negative of } f(3) \text{)} \]

3. Which of the following is a factor of the cubic expression \( x^3 - 3x^2 - 64x - 60 \)?

(1) \( (x - 5) \)  
(2) \( (x + 10) \)  
(3) \( (x + 1) \)  
(4) \( (x - 12) \)

\[ \text{Graph the function} \]
\[ \text{in } Y_1 \text{, and find an } X \text{-int.} \]
\[ \text{X-ints at } x = -6 \Rightarrow (x + 6) \]
\[ x = -1 \Rightarrow (x + 1) \]
\[ x = 10 \Rightarrow (x - 10) \]
5. Which graph has the following characteristics?
- $4^{th}$ degree
- As $x \to -\infty$, $f(x) \to -\infty$ DOWN TO LEFT
- As $x \to \infty$, $f(x) \to \infty$ DOWN TO RIGHT

(1) \hspace{1cm} (3)

(2) \hspace{1cm} (4)

6. Divide: $(2x^2 + 2x + 5) + (x + 2)$

\(2x - 2 + \frac{9}{x+2}\)  
\(2x + 6 + \frac{17}{x+2}\)

7. Algebraically determine whether the function $f(x) = x^4 + 3x^2 - 12$ is even, odd, or neither.

$f(-x) = (-x)^4 + 3(-x)^2 - 12 = x^4 + 3x^2 - 12$ so EVEN FUNCTION $f(-x) = f(x)$

8. Find the third term in the expansion of $(x - 2y)^4$

\[\binom{4}{3} x^{4-3} (-2y)^3 = 4 \cdot x \cdot 8 \cdot y^3 = 32xy^3\]
9. Given: \( p(x) = x^3 + 3x^2 - 4x - 12 \), find all values of \( x \) that satisfy the equation \( p(x) = 0 \). (Only an algebraic solution will be acceptable.)

\[
\begin{align*}
\quad & x^3 + 3x^2 - 4x - 12 = 0 \\
\quad & x^2(x + 3) - 4(x + 3) = 0 \\
\quad & (x^2 - 4)(x + 3) = 0 \\
\quad & (x + 2)(x - 2)(x + 3) = 0 \\
\quad & x = -2, 2, -3
\end{align*}
\]

10. The graph of a polynomial is shown below.

What is the degree of the this polynomial function? \( \underline{6} \)th DEGREE

As \( x \to \infty \), then \( f(x) \to +\infty \).  
Move (left)

As \( x \to -\infty \), then \( f(x) \to +\infty \).  
Move (right)

Is this function even, odd, or neither? Why? \underline{NEITHER}  
Does not have y-axis symmetry (even)  
Does not have it sym thru the origin (odd)

11. Find an equation of the following polynomial function.

\[
\begin{align*}
y &= a(x + 2)^2(x - 4)(x - 7) \\
1 &= a(0 + 2)^2(0 - 4)(0 - 7) \\
1 &= a(4)(-4)(-7) \\
a &= \frac{1}{112}
\end{align*}
\]

\[
y = \frac{1}{112}(x + 2)^2(x - 4)(x - 7)
\]