Unit 2 Review Sheet

1. The graph of the quadratic function \( y = ax^2 + bx + c \) is shown below. Which of the following must be true about the values of \( a, b, \) and \( c? \)

   (3) \( a \cdot c < 0 \)
   (4) \( b \cdot c < 0 \)

2. For the function \( f(x) = x^2 - 2x - 15, \) over which of the following intervals is \( f(x) > 0 \) always?

   (2) \( x < 0 \)
   (4) \( x < -3 \) or \( x > 5 \)

3. Selected values of a quadratic function are shown below. Which of the following values of \( x \) represents an \( x \)-intercept of the function?

   (3) 6
   (4) 10

4. The function \( y = -\frac{1}{2}(x-6)^2 + 17 \) is strictly decreasing over which of the following intervals

   (1) \( x > 6 \)
   (2) \( x < 6 \)
5. What is the x-coordinate of the turning point of the parabola \( y = 5x^2 + 27x - 3 \)?

\[
\begin{align*}
(1) \ x &= -2.7 \\
(2) \ x &= -1.8 \\
(3) \ x &= 5.4 \\
(4) \ x &= 7.2 \\
\end{align*}
\]

\[
X = \frac{-b}{2a} = \frac{-27}{2(5)} = \frac{-27}{10} = -2.7
\]

or graph \( y \rightarrow \text{2nd frame MIN} \):

6. The parabola \( y = 3x^2 - 24x + 55 \) can be written in the form

\[
\begin{align*}
(1) \ y &= 3(x-2)^2 + 2 \\
(2) \ y &= 3(x-8)^2 + 55 \\
(3) \ y &= 3(x+2)^2 - 11 \\
(4) \ y &= 3(x-4)^2 + 7
\end{align*}
\]

\[\text{Find the vertex on the graph: } V(4,7)\]

7. A cable hangs in a parabolic shape above a level surface between two poles such that its height above the surface is given by the equation \( y = 0.001x^2 - 1.24x + 16 \), where \( y \) is the cable's height above the surface, in feet, and \( x \) is the horizontal distance from one of the poles. According to this model, which of the following represents the lowest height the cable is above the surface?

\[
\begin{align*}
(1) \ 8.9 \text{ feet} \\
(2) \ 10.1 \text{ feet} \\
(3) \ 11.3 \text{ feet} \\
(4) \ 12.2 \text{ feet}
\end{align*}
\]

\[\text{Minimum } y \text{ value: } y_{\text{min}} = 12.186\]

8. A parabola has an axis of symmetry \( x = -2 \) and passes through the point \((-5, 6)\). Find another point that lies on the graph of the parabola.

\[\text{Another point: } (1,6)\]

9. Let the graph of \( g \) be a vertical stretch by a factor of 4 and a reflection in the x-axis of the graph of \( f(x) = x^2 - 3 \). Write a rule for \( g \).

\[y = 4f(x) \Rightarrow y = -4(x^2-3) \text{ or } y = -4x^2 + 12\]
10. Identify the focus, directrix, and axis of symmetry of \( f(x) = \frac{1}{16}x^2 \).

- **F**: \((0, 4)\)
- **D**: \(y = -4\)
- **V**: \((0, 0)\)
- **AOS**: \(x = 0\) (Y-axis)
- \(p = 4\)
- \(\frac{a}{16} = \frac{1}{4p} \Rightarrow so \ p = 4\)

11. Write the equation of the parabola.

**a)**

\[
p = 1
\]

\[
a = \frac{1}{4p} = \frac{1}{4(1)} = \frac{1}{4}
\]

\[
y = \frac{1}{4}(x)^2 + 2
\]

**b)**

12. A bridge builder plans to construct a cable suspension bridge in your town. The cable being used will form a curve modeled by the equation \(h(x) = 3x^2 - 6x + 200\), where \(x\) represents the length of cable used (in feet) and \(h(x)\) represents the height of the cable (in feet). At what height will the cable hang closest to the bridge deck?

- **Factored Form**:

\[
y = a(x + 2)(x - 6)
\]

- **Vertex**

\[
y = \frac{-1}{4}(x + 2)(x - 6)
\]

**So after 1 ft the cable will hang at a height of 197 ft (closest to roadway)
13. Graph \( f(x) = 3(x + 1)^2 - 3 \). Label the vertex and axis of symmetry. Describe where the function is increasing and decreasing.

\[
V(-1, -3) \quad \text{INCREASING: } (-1, \infty) \\
AOS: x = -1 \quad \text{DECREASING: } (-\infty, -1)
\]

14. Graph \( g(x) = -2x^2 - 4x + 3 \). Label the vertex and axis of symmetry. Describe where the function is increasing and decreasing.

\[
V(-1, 5) \\
AOS: x = -1 \\
\text{INCREASING: } (-\infty, -1) \\
\text{DECREASING: } (-1, \infty)
\]
15. The tables show the number of toy bears a toy manufacturer can sell.

<table>
<thead>
<tr>
<th>Price (dollars), $x$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bears sold (in thousands), $y$</td>
<td>84</td>
<td>96</td>
<td>100</td>
<td>96</td>
<td>84</td>
</tr>
</tbody>
</table>

a) Write a quadratic regression equation, rounding coefficients to the nearest hundredth.

\[
\text{STAT \rightarrow L1, L2 \rightarrow Stats \rightarrow Quad Reg.} \\
\text{\hspace{1cm} } y = ax^2 + bx + c \\
\text{\hspace{1cm} } a = -4 \quad b = 40 \quad c = 0
\]

\[
y = -4x^2 + 40x
\]

b) Using your equation found in part (a), determine how many bears the manufacturer will sell if it charges $9 for each bear.

\[
\text{This is an x-value} \\
\text{So plug in } x = 9 \text{ to your equation} \\
\text{(or look on the table of values)}
\]

\[
x = 9 \quad y = 36 \quad \text{(thousands)}
\]

16. Your class council determined that its profit from the upcoming homecoming dance is directly related to the ticket price for the dance. Looking at past dances, the council determined that the profit $p$ can be modeled by the function $p(t) = -12t^2 + 480t + 30$, where $t$ represents the price of each ticket. What should be the price of a ticket to the homecoming dance to maximize the council’s profit?

\[
\text{2nd Trace Max:} \\
V(20, 4830)
\]

So when the price is $20 for each ticket the profit is $4830.