The average rate of change ("slope") is an exceptionally important concept in mathematics because it gives us a way to quantify how fast a function changes on average over a certain domain interval. Although we used its formula in the last exercise, we state it formally here:

### Average Rate of Change ("Slope")

For a function over the domain interval \( a \leq x \leq b \), the function's average rate of change is calculated by:

\[
\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b) - f(a)}{b - a}
\]

A linear function will have a **CONSTANT** rate of change.

Linear functions come in a variety of forms. The two shown below have been introduced in Common Core Algebra I and Common Core Geometry.

### Two Common Forms of a Line

**Slope-Intercept:** \( y = mx + b \)

**Point-Slope:** \( y - y_1 = m(x - x_1) \)

where \( m \) is the slope (or average rate of change) of the line and \((x_1, y_1)\) represents one point on the line.

#### Transformations of Linear Functions:

- \( -f(x) = \) ______________________________

- \( f(-x) = \) ______________________________

- \( f(x) + k = \) ______________________________

- \( f(x) - k = \) ______________________________

- \( f(x + k) = \) ______________________________

- \( f(x - k) = \) ______________________________

- \( kf(x) = \) ______________________________

- \( \frac{1}{k} f(x) = \) ______________________________

- \( f(kx) = \) ______________________________

- \( f\left(\frac{1}{k}x\right) = \) ______________________________
Solving Systems of Linear Equations

Systems of equations, or more than one equation, arise frequently in mathematics. To solve a system means to find all sets of values that simultaneously make all equations true.

Consider the three-by-three system of linear equations shown below. Each equation is numbered to help you keep track of the manipulations.

\[
\begin{align*}
2x + y + z &= 15 \quad (1) \\
6x - 3y - z &= 35 \quad (2) \\
-4x + 4y - z &= -14 \quad (3)
\end{align*}
\]

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

(b) Use this new two-by-two system to solve the three-by-three.

First multiply the second equation by 4:

\[
4(-2x + 5y) = 4(1) \Rightarrow -8x + 20y = 4
\]

This is an effective strategy because in both cases the \(z\)-variable was eliminated from the equations.

\[
\begin{align*}
8x - 2y &= 50 \\
-8x + 20y &= 4 \\
18y &= 54 \\
y &= 3
\end{align*}
\]

\[
\begin{align*}
8x - 2(3) &= 50 \\
8x - 6 &= 50 \\
8x &= 56 \\
x &= 7
\end{align*}
\]

Exercise #2: Consider the system of equations shown below. Answer the following questions based on the system.

\[
\begin{align*}
4x + y - 3z &= -6 \\
-2x + 4y + 2z &= 38 \\
5x - y - 7z &= -19
\end{align*}
\]

Multiply the first equation by 2:

\[
2(9x - 10z) = 2(-25) \\
18x - 20z = -50
\]

The variable \(y\) will be easiest to eliminate first because it has a coefficient of 1 in the first equation.

\[
\begin{align*}
(1) + (3): \\
4x + y - 3z &= -6 \\
5x - y - 7z &= -19 \\
9x - 10z &= -25
\end{align*}
\]

\[
\begin{align*}
-4 \times (1) + (2): \\
-16x - 4y + 12z &= 24 \\
-2x + 4y + 2z &= 38 \\
-18x + 14z &= 62
\end{align*}
\]

\[
\begin{align*}
18x - 20z &= -50 \\
-18x + 14z &= 62 \\
-6z &= 12 \\
z &= -2
\end{align*}
\]

\[
\begin{align*}
18x - 20(-2) &= -50 \\
18x + 40 &= -50 \\
18x &= -90 \\
x &= -5
\end{align*}
\]

\[
\begin{align*}
4(-5) + y - 3(-2) &= -6 \\
-20 + y + 6 &= -6 \\
y - 14 &= -6 \\
y &= 8
\end{align*}
\]
Unit 2 – Quadratic Functions Summary

### QUADRATIC FUNCTIONS

Any function of the form $f(x) = ax^2 + bx + c$ where the leading coefficient, $a$, is not zero.

a) Standard Form: $f(x) = ax^2 + bx + c$ useful for_________________________________________

b) Vertex Form: $f(x) = a(x-h)^2 + k$ useful for_________________________________________

c) Factored Form: $f(x) = a(x-r_1)(x-r_2)$ useful for_________________________________________

Definitions:

1. Turning Point (Vertex):___________________________________________________

2. Axis of Symmetry:_______________________________________________________

3. Focus:_________________________________________________________________

4. Directrix:_______________________________________________________________

5. Roots:_________________________________________________________________

Consider the simplest of all quadratic functions $y = x^2$.

(a) Create a table of values to plot this function over the domain interval $-3 \leq x \leq 3$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^2$</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

(b) Sketch a graph of this function on the grid to the right.

(c) State the coordinates of the turning point of this parabola.

(d) State the equation of this parabola’s axis of symmetry.

(e) Over what interval is this function increasing? Decreasing?

How is $x = y^2$ different? Sketch the graph to the right.

State the Domain:_______________________________

State the Range:_______________________________
Example: Use the formula \( x = \frac{-b}{2a} \) to find the turning points for each of the following quadratic functions.

(a) \( f(x) = 2x^2 - 12x + 7 \)

\[
x = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3
\]
\[
y = 2(3)^2 - 12(3) + 7 = -11
\]
Turning point: \((3, -11)\)

(b) \( g(x) = -\frac{1}{4}x^2 + 5x - 20 \)

\[
x = \frac{-5}{-\frac{1}{4}} = \frac{5}{-\frac{1}{4}} = 10
\]
\[
y = -\frac{1}{4}(10)^2 + 5(10) - 20 = 5
\]
Turning point: \((10, 5)\)

(*Always verify by graphing the function on your calculator!)

(*Check on your table of values!)

Rewrite in Vertex form:_____________________
Rewrite in Vertex form:_____________________

Describe the transformation from \( y = x^2 \)
Describe the transformation from \( y = x^2 \)

___________________________

___________________________

Quadratic Regressions:

1. Hit STAT→EDIT. Enter data into L1, L2
2. Hit STAT→CALC. Select option #5 QuadReg
3. Scroll down to Calculate.
4. Copy down the equation. Round coefficients as specified.

Example: A baseball is thrown up in the air. The table shows the heights \( y \) (in feet) of the baseball after \( x \) seconds.

<table>
<thead>
<tr>
<th>Time, ( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball height, ( y )</td>
<td>6</td>
<td>22</td>
<td>22</td>
<td>6</td>
</tr>
</tbody>
</table>

a) Write an equation for the path of the baseball. ________________________________

b) Find the height of the baseball after 5 seconds._______________________________

c) What is the maximum height of the baseball? _________________________________

d) When does the baseball reach its maximum height? _____________________________

Focus & Directrix of a Parabola
A parabola is the collection of all points equidistant from a fixed point (known as its focus) and a fixed line (known as its directrix).

Consider parabola that is the collection of all points equidistant from the point \((0, 8)\) and the line \( y = 2 \).
Example 1:
Determine the equation of the parabola whose focus is the point \((4, 1)\) and whose directrix is the horizontal line \(y = -3\). First, draw a diagram that shows the parabola, then carefully use the formula \(a = \frac{1}{4p}\) to derive its equation.

Example 2:
Given the equation of a parabola to be \(y = \frac{1}{12} x^2 + 1\), determine the following and sketch the graph labeling each of the parameters listed below:

- a) Vertex __________________________
- b) “a” value ______________________
- c) “p” value ______________________
- d) Focus __________________________
- e) Directrix ________________________