Unit 1: Review Sheet

1. Write an equation of the line and interpret the slope and y-intercept.

   a) Child Growth Rate
   
   **Equation:** \( y = 3x + 12 \)
   
   **Slope:** \( m = \frac{30 - 15}{6 - 1} = \frac{15}{5} = 3 \)
   
   **y-intercept:** \( (0, 12) \)
   
   Initial birth ht. is 12”.

   b) Home Phone Sales
   
   **Equation:** \( y = -100x + 700 \)
   
   **Slope:** \( m = \frac{200 - 600}{5 - 1} = -\frac{400}{4} = -100 \)
   
   **y-intercept:** \( (0, 700) \)
   
   Initial sales in millions.

   c) Walking
   
   **Equation:** \( y = 3 \)
   
   **Slope:** \( m = \frac{3 - 3}{4 - 2} = 0 \)
   
   **y-intercept:** \( (0, 3) \)

2. If \( f(x) = x^2 + 4 \) and \( g(x) = \sqrt{1 - x} \), what is the value of \( f(g(-3)) \)?
   
   (1) \( 2i\sqrt{3} \)
   
   (2) \( 2 \)
   
   (3) \( 8 \)
   
   (4) \( 13 \)

3. If \( f(x) = x^2 - 5 \) and \( g(x) = 6x \), then \( g(f(x)) \) is equal to

   (1) \( 6x^3 - 30x \)
   
   (2) \( 6x^2 - 30 \)
   
   (3) \( 36x^2 - 5 \)
   
   (4) \( x^2 + 6x - 5 \)
4. For each function below, find $f^{-1}(x)$.

\[f(x) = 3x + 5\]
\[g = 3x + 5\]
\[x = \frac{3y + 5}{3}\]
\[x - 5 = \frac{3y}{3}\]
\[f^{-1}(x) = \frac{x - 5}{3}\]

\[f(x) = x^2 + 2\]
\[g = x^2 + 2\]
\[x = \frac{y^2 + 2}{2}\]
\[\sqrt{x - 2} = \frac{y^2}{2}\]
\[f^{-1}(x) = \sqrt{\frac{x - 2}{2}}\]

\[f(x) = \frac{x - 1}{2x + 5}\]
\[2xy - y = -5x - 1\]
\[g(x) = \frac{x - 1}{2x + 5}\]
\[\frac{2x}{2x + 1} = \frac{-5x - 1}{(2x + 1)(2x - 1)}\]
\[g^{-1}(x) = \frac{-5x - 1}{2x - 1}\]

Describe how $g(x)$ is transformed from $f(x)$. Then graph $g(x)$ and state its domain and range, and whether it is even, odd, or neither.

5. \[f(x) = x\]
   a) \[g(x) = x - 5\]
   
   ![Graph of shifted down 5]
   
   Domain: $(-\infty, \infty)$
   Range: $(-\infty, \infty)$
   Even/Odd: Neither
   (No y-axis symmetry or pt symmetry over origin)

   b) \[g(x) = |x| + 2\]
   
   ![Graph of shifted up 2]
   
   Domain: $(-\infty, \infty)$
   Range: $[2, \infty)$
   Even/Odd: Even
   (y-axis symmetry)

   c) \[g(x) = \frac{-1}{3}x^2\]
   
   ![Graph of reflected over x-axis vertically compressed by factor of 1/3]
   
   Domain: $(-\infty, \infty)$
   Range: $(-\infty, 0]$ (Don't write this backwards)
   Even/Odd: Even
   (y-axis symmetry)

6. Given the function $f(x) = x^2 - 2x + 7$, what is its average rate of change over the interval $3 \leq x \leq 11$?

\[
m = \frac{f(11) - f(3)}{11 - 3} = \frac{106 - 10}{8} = \frac{96}{8} = 12
\]
7. Let \( f(x) \) represent a function. Explain what happens to the graph of \( f(x) \) under the following transformations:

a) \( f(-x) \) \text{ reflected over the } x-\text{axis.}

b) \( f(x-5) \) \text{ reflected over the } y-\text{axis then shifted down 5.}

c) \( f(x+2) \) \text{ shifted left } 2.

d) \( 2f(x) \) \text{ vertically stretched by a factor of 2.}

e) \( f(x+1)-1 \) \text{ shifted left 1 and down 1.}

8. Given the function \( f(x) = x^2 - 2x + 7 \), what is its average rate of change over the interval \( 3 \leq x \leq 11 \)?

3 \hspace{1cm} (1) 8 \hspace{1cm} (2) -5 \hspace{1cm} (3) 12 \hspace{1cm} (4) -7

\( (\text{Same as } \# 6) \)

9. The table below shows the amount of fuel left in your RV while driving. State the function that models the data. Estimate the amount of fuel left in your tank after driving for 90 minutes.

<table>
<thead>
<tr>
<th>Time (minutes), ( x )</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel left (gallons), ( y )</td>
<td>18</td>
<td>16</td>
<td>14</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

or use the equation:

\[ m = \frac{16-18}{20-0} = \frac{-2}{20} = \frac{-1}{10} \text{ or } -0.1 \]

\[ b = 18 \]

\[ y = -\frac{1}{10}x + 18 \]

plug in 90 for \( x \)

\[ y = 9 \]
10. Selected values of a linear function \( f(x) \) are given in the table below. Find the value of \( k \). Explain how you found your answer.

\[
\begin{array}{c|c|c|c|c|c}
  x & -8 & -2 & 4 & 12 & 14 & 18 \\
  f(x) & -33 & -12 & k & 37 & 44 & 58 \\
\end{array}
\]

\[
y = mx + b
\]
\[
a = 3.5
\]
\[
b = -5
\]
\[
r = 1
\]

\[
y = 3.5x - 5
\]

Type into \( Y_1 \) then go to table of values at \( x = 4 \) and find \( y \).

\[
\begin{array}{c|c|c|c}
  x & 4 & 9 \\
  y & & s0(k = 9) \\
\end{array}
\]

11. Write an equation for the line passing through the points \((-5, 15)\) and \((20, 25)\). Show how you arrived at your answer.

\[
m = \frac{25-15}{20-(-5)} = \frac{10}{25} = \frac{2}{5} = .4
\]

\[
y = mx + b
\]

\[
y = .4x + b
\]

\[
y = .4x + b
\]

Now plug in either pt \( x,y \):

\[
ym = .4 \cdot 20 + b = 8 + b
\]

\[
y = 8 + b
\]

\[
y = 25
\]

\[
y = 8 + b
\]

\[
y = 17 = b
\]

\[
ym = .4x + 17
\]

12. After a recent Arlington High School basketball game, traffic was exiting the parking lot at a constant rate of 28 cars per minute. The parking lot started with 922 cars.

a) How many cars are still in the parking lot after 10 minutes?

plug in \( x = 10 \)

\[
y = -28(10) + 922
\]

\[
y = 642
\]

(642 cars left after 10 min)

b) After 25 minutes the rate at which the cars leave rises to 34 cars per minute. How many total minutes does it take for the parking lot to completely clear? Round to the nearest minute. Show your analysis.

First 25 min:

\[
y = -28(25) + 922
\]

\[
y = 222
\]

After first 25 min:

\[
\begin{aligned}
\text{Starting with } & 222 \text{ cars left, and leaving at rate of } 34 \text{ cars/min} \\
& m = -34 \\
& b = 222 \\
& o = -34x + 222 + 34x + 222 \\
& x = 6.526
\end{aligned}
\]
13. Solve the following system of equations algebraically.

\[ \begin{align*}
3x - 5y + 2z &= -5 \\
5x + y + 6z &= 33 \\
-2x + 10y - 3z &= 40
\end{align*} \]

Combine (1) + (2):
\[ -3(3x - 5y + 2z = -5) \Rightarrow -9x + 15y - 6z = 15 \\
5x + y + 6z = 33 \Rightarrow 5x + y + 6z = 33 \]
Combine (3) + (2):
\[ 2x + 10y - 32 = 40 \Rightarrow 2x + 10y = 80 \]
Combine (4) + (5):
\[ 1x + 21y = 113 \]

Now combine (4) + (5):
\[ -4x + 16y = 48 \Rightarrow -4x + 16y = 48 \\
-x + 2y = 113 \Rightarrow -4x + 8y = 452 \]
\[ \frac{100y = 500}{100} \Rightarrow y = 5 \]
\[ \frac{100z = 10}{100} \Rightarrow z = -12 \]
\[ x = 8 \]

4. Three orders are placed at a pizza shop. Two small pizzas, a liter of soda, and a salad cost $14; one small pizza, a liter of soda, and three salads cost $15; and three small pizzas, a liter of soda, and two salads cost $22. How much does each item cost?

Let:
\[ p = \# \text{ of pizzas ordered} \]
\[ l = \# \text{ of liters of soda ordered} \]
\[ s = \# \text{ of salads ordered} \]

Combine (1) + (2):
\[ 2p + l + s = 14.00 \]
\[ 1p + l + 3s = 15.00 \]
\[ 3p + l + 2s = 22.00 \]

Combine (1) + (3):
\[ 2p + l + s = 14.00 \Rightarrow 2p + l + s = 14.00 \]
\[ 2p + l + s = 14.00 \Rightarrow 2p + l + s = 14.00 \]
\[ 3p + l + 2s = 22.00 \Rightarrow -3p - l - 2s = -22.00 \]

Combine (4) + (5):
\[ 1p - 2s = -1.00 \]
\[ 1p - 1s = -8.00 \]
\[ -3s = -9.00 \]
\[ s = 3 \]
\[ \frac{-3}{3} \]

Combine (1) + (4):
\[ 2p + l + s = 14.00 \]
\[ 10 + l + 3 = 14 \]
\[ 13 + l = 14 \]
\[ l = 1 \]
\[ x = 5 \]
\[ p = 5 \]
\[ l = 1 \]