Sequences Quiz

Part I: Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

1. What is the formula for the $n$th term of sequence $B$ shown below?

   $B = 8, 12, 18, 27...$

   (1) $b_n = 4 + 4n$  (2) $b_n = 8\left(\frac{3}{2}\right)^n$  (3) $b_n = 8 + 4n$  (4) $b_n = 8\left(\frac{3}{2}\right)^{n-1}$

2. What is the common ratio of the geometric sequence shown below?

   $-2, 4, -8, 16$

   (1) $-\frac{1}{2}$  (2) 2  (3) -2  (4) -6

3. Which of the following is an example of a geometric sequence?

   (1) $\{0, 4n, 8n, 12n, \ldots\}$  (3) $\{n + 1, n + 5, n + 9, n + 13, \ldots\}$

   (2) $\{n, 4n, 16n, 64n, \ldots\}$  (4) $\{n + 4, n + 16, n + 64, n + 256, \ldots\}$

4. What is the common ratio of the geometric sequence whose first term is 64 and fifth term is 324?

   (1) $\frac{3}{4}$  (2) $\frac{3}{2}$  (3) $\frac{2}{3}$  (4) 65

5. The eighth and tenth terms of the sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term cannot be

   (1) -82  (2) -80  (3) 80  (4) 82

   (GEO with negative "r")  (GEO with + "r")  (APRTH \(d = 18\))
Part II: Answer all questions in this part. Each correct answer will receive 4 credits. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

6. List a sequence that has 3 arithmetic means between 8 and 24.

\[3, \frac{3 + 4d}{2}, \frac{3 + 4d}{2}, \frac{3 + 4d}{2}, 24\]

\[\frac{3 + 4d}{2} = 16\]

\[\frac{3 + 4d}{2} = \frac{2(2 - d)}{1}\]

\[d = \frac{-21}{4} = -5.25\]

7. Given the following geometric sequence below, find the sum of the first 10 terms:

\[3, -6, 12, -24, \ldots\]

\[a_1 = 3\]

\[r = -2\]

\[S_{10} = \sum_{n=1}^{10} (3(-2)^{n-1}) = \frac{3(-2)^1 (1 - 2^{10})}{1 - 2} = -1023\]

8. Find the sum of an infinite number of terms of the geometric series whose common ratio is \(\frac{1}{2}\) and \(a_1 = 8\).

\[\sum_{n=1}^{\infty} (8(\frac{1}{2})^{n-1}) = \sqrt{16} = 4\]

Part III: Answer all questions in this part. Each correct answer will receive 4 credits. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

9. If \(x + 4, 2x + 5\) and \(4x + 3\) represent the first three terms of an arithmetic sequence, then find the value of \(x\). What is the fourth term?

\[\frac{15}{1}\]

\[\frac{1}{x+4}\]

\[\frac{1}{2x+5}\]

\[\frac{1}{4x+3}\]

\[a_4 = \frac{19}{3}\]

* The common diff. must be the same!

\[\text{So } (2x + 5) - (x + 4) \text{ must } = (4x + 3) - (2x + 5)\]

\[x + 1 = 2x - 2\]

\[x + 2 = -x + 2\]

\[3 = x\]

\[x = 3\]

NOW, plug back in to get values above.
10. If $a_4 = -4$ and $a_8 = 12$ are given terms of an arithmetic sequence, answer the following:

(a) Write the $n^{th}$ term formula for the given arithmetic sequence. [2 pts]

\[
\begin{align*}
16 & -12 & 8 & -4 & 0 & 4 & 8 & 12 \\
-4 & + 4d = 12 \\
\frac{d}{4} & = 4 \\
q_4 & = -16 + (n-1)(4) \\
\end{align*}
\]

b) What is the SUM of the first 50 terms? [2 pt]

\[
\sum_{n=1}^{50} (-16 + (n-1)(4)) = 4100
\]

Part IV: Answer all questions in this part. Each correct answer will receive 6 credits. For all questions in this part, a correct numerical answer without work shown will receive only 1 credit.

11. Caitlin has a movie rental card worth $75. After she rents the first movie, the card's value is $71. After she rents the second movie, its value is $67. After she rents the third movie, the card is worth $63.

Assuming the pattern continues, write an explicit formula to define $A(n)$, the amount of money on the rental card after $n$th rentals.

\[
A_1 = 75, \qquad d = -4
\]

\[
A_n = 75 + (n-1)(-4)
\]

Now find an equivalent way to express $A(n)$ using a recursive formula.

\[
A_1 = 75, \quad A_n = A_{n-1} - 4
\]

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Justify how you arrived at your answer.

\[
\begin{align*}
75 + (n-1)(-4) & = 0 \\
75 - 4n + 4 & = 0 \\
79 - 4n & = 0 \\
\frac{79}{4} & = \frac{4n}{4} \\
\end{align*}
\]

So there are 19 terms, but the 1st term isn't a rental so therefore not enough for another one.

\[
\begin{align*}
\frac{79}{4} & = \frac{4n}{4} \\
\end{align*}
\]

\[
\begin{align*}
79 & = 4n \\
\frac{79}{4} & = \frac{4n}{4} \\
\end{align*}
\]

\[
\begin{align*}
79 - 4n & = 0 \\
\end{align*}
\]

\[
\begin{align*}
A_n & = A_{n-1} - 4 \\
\end{align*}
\]

\[
\begin{align*}
A_1 & = 75 \\
A_n & = A_{n-1} - 4 \\
\end{align*}
\]

\[
\begin{align*}
75, 71, 67, 63, 59, 55, 51, 47, 43, 39, 35, 31, 27, 23, 19, 15, 11, 7, 3, \times \\
\end{align*}
\]

\[
\begin{align*}
18^{th} \text{ rental} \\
\end{align*}
\]

\[
\begin{align*}
18 \text{ weeks} \\
\end{align*}
\]

\[
\begin{align*}
\text{you get 18 weeks in row her rentals} \\
\end{align*}
\]

\[
\begin{align*}
\text{Not enough for} \\
\end{align*}
\]

\[
\begin{align*}
\text{another one} \\
\end{align*}
\]

\[
\begin{align*}
\text{10} \\
\end{align*}
\]