Name: **Answers**

**Rational Functions**

**Vertical Asymptote vs “Hole” in the graph**

Given: \( y = \frac{x^2 - 2x - 8}{x^2 - 1} = \frac{(x-4)(x+2)}{(x+1)(x-1)} \)  

1. Find the horizontal asymptote, if any.  
2. Find the vertical asymptote(s), if any.  
3. Find the \( x \) intercept(s), if any.  
4. Find the \( y \) intercept, if any.  

(Check your answers with the graph!!!)

\[ y = \frac{x^2}{x^2} \Rightarrow y = 1 \]

\( x = -1, \; x = 1 \)

\( x = 4, \; x = -2 \) or \( (4,0) (6,0) \)

\( y = \frac{-8}{1} = 8 \)  

\( (0,8) \)

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Now list the same pieces of information for this rational function.

Given: \( y = \frac{x^2 - 5x - 6}{x^2 - 1} = \frac{(x-6)(x+1)}{(x+1)(x-1)} \)

\( \Rightarrow y = \frac{(x-6)}{(x-1)} \) \( \quad \text{HOLE at } x = -1 \)

\( y = \frac{x^2}{x-1} \Rightarrow y = 1 \)

\( x = 1, \; x = -1 \)

\( x = 6, \; x = -1 \) or \( (6,0) (-1,0) \)

\( y = \frac{-6}{1} = 6 \)  

\( (0,6) \)

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Do all of your answers agree with the graph?  
Which answers seem different?  **NO!**  **NO VERT ASY @ X = -1**  
**NO X INT @ X = -1**

What creates this difference?  **SIMILAR FACTORS THAT CANCEL!**

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How do you know when there is a “hole” vs when there is a Vertical Asymptote?  
If \**SIMILAR FACTORS CAN CANCEL FROM TOP + BOTTOM**  
then \**HERE IS A HOLE (POINT OF REMOVABLE DISCONTINUITY)**  

How do you find the coordinates of any “hole” in the graph?  
**USE THE X-VALUE FROM THE CANCELLED FACTOR AND PLUG INTO THE REDUCED (SIMPLIFIED) FORM OF THE EQUATION TO FIND THE Y-VALUE.**
Given: \( y = \frac{3x^2 - 3x - 18}{2x^3 - 18} \)

1. Find the horizontal asymptote, if any.
2. Find the vertical asymptote(s), if any.
3. Find the x intercept(s), if any.
4. Find the y intercept, if any.
5. Find the coordinates of the "hole", if any

\[ \Rightarrow y = \frac{3(x+2)}{2(x+3)} \]

(Check your answers with the graph!!!)

\[ y = \frac{3x^2}{2x^2} \Rightarrow y = \frac{3}{2} \]

\[ x = -3 \]

\[ x = -2 \text{ or } (-2,0) \]

\[ y = \frac{11}{8} \Rightarrow y = 1 \quad (0,1) \]

\[ (3, \frac{15}{4}) \text{ or } (3, \frac{15}{4}) \]

\[ \Rightarrow \text{ use } y = \frac{3(x+2)}{2(x+3)} \text{ and plug in } x = 3: \quad y = \frac{3(3+2)}{2(3+3)} \]

\[ y = \frac{15}{12} \text{ so } (3, \frac{15}{12}) \text{ or } (3, \frac{3}{4}) \]

Given: \( y = \frac{2x+4}{x^2-16} = \frac{2x+2}{(x+4)(x-4)} \)

1. Find the horizontal asymptote, if any.
2. Find the vertical asymptote(s), if any.
3. Find the x intercept(s), if any.
4. Find the y intercept, if any.
5. Find the coordinates of the "hole", if any

\[ y = \frac{2x}{x-4} \Rightarrow y = \frac{2}{1} \Rightarrow y = 0 \]

\[ x = -4, \quad x = 4 \]

\[ x = -2 \text{ or } (-2,0) \]

\[ y = \frac{4}{16} \Rightarrow y = -\frac{1}{4} \quad (0, -\frac{1}{4}) \]

NONE!

Given: \( y = \frac{\sqrt{x}}{x^2+9} \)

1. Find the horizontal asymptote, if any.
2. Find the vertical asymptote(s), if any.
3. Find the x intercept(s), if any.
4. Find the y intercept, if any.
5. Find the coordinates of the "hole", if any

\[ x^2 + 9 \text{ CAN NEVER = 0 FOR ANY REAL NUMBER.} \]

\[ x^2 + 9 = 0 \rightarrow x = \pm 3i \]

\[ x = \pm 3i \text{ SO NO REAL VERT ASYMPTOTES!} \]

\[ y = \frac{x}{x^2} = \frac{1}{x} \rightarrow 0 \quad y = 0 \]

NONE! (since \( x^2 + 9 \neq 0 \))

\[ x = 5 \text{ or } (5, 0) \]

\[ y = \frac{-5}{9} \text{ or } (0, -\frac{5}{9}) \]

NONE!