1. The graph of \( y = x^3 - x^2 - 6x \) is shown below.

* Each root is a single zero since it travels straight through the x-axis...

(No "bouncing off" or "flattening out")

Which set lists all the real solutions of \( f(x) = 0 \)?

(1) \{ -2, 0 \}  (2) \{ -2, 3 \}  (3) \{ -2, 0, 3 \}  (4) \{ 0 \}

Now, use these roots to create factors for this polynomial function.

Root 1: \( x = -2 \)  Factor is \( (x + 2) \)  Because it's \( (x - (-2)) \)

Root 2: \( x = 0 \)  Factor is \( (x - 0) \)  (or just \( x \))

Root 3: \( x = 3 \)  Factor is \( (x - 3) \)

Now write the equation in factored form: \( y = (x + 2)(x)(x - 3) \)

(Graph the equation to double check.)

2. Write an equation of the polynomial of smallest degree whose graph is shown below.

Initial attempt
\[ x = -5 \Rightarrow (x + 5) \text{ factor} \]
\[ x = -2 \Rightarrow (x + 2) \text{ factor} \]
\[ x = 6 \Rightarrow (x - 6) \text{ factor} \]
So therefore:
\[ y = (x + 5)(x + 2)(x - 6) \]

However, when you graph this, the y-int does not line up.

Are they the same? (Graph your answer to double check!)

***Did you account for the "a" value?  Fix this by using the y-int of \( (0, -4) \) to plug in for \( (x, y) \) in the equation to solve for the "a" coefficient.

\[ a = \frac{15}{-6} = \frac{1}{5} \]  \[ y = \frac{15}{-6} (x + 5)(x + 2)(x - 6) \]
**Multiplicity of Roots**

Are the functions \( p(x) = x(x-3)^2 \), \( q(x) = x(x-3)^3 \), and \( r(x) = x^2(x-3)^3 \) all equivalent? Justify your answer.

Let's summarize what the graphs of Polynomial functions look like when there is multiplicity with their zeros!

<table>
<thead>
<tr>
<th>If the linear factor of the polynomial is raised to ....</th>
<th>Then the graph of the polynomial at that zero will ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>the first power</td>
<td>&quot;LINEAR OR SINGLE ROOT&quot;</td>
</tr>
<tr>
<td>an even power</td>
<td></td>
</tr>
<tr>
<td>(we usually use 2 as the basic even power)</td>
<td>&quot;DOUBLE ROOT&quot;</td>
</tr>
<tr>
<td>an odd power ( &gt; 1 )</td>
<td>&quot;TRIPLE ROOT&quot;</td>
</tr>
</tbody>
</table>

For example:

- For \( p(x) = x(x-3)^2 \), the graph will bounce off the x-axis at \( x = 3 \).
- For \( q(x) = x(x-3)^3 \), the graph will flatten out at \( x = 3 \).
- For \( r(x) = x^2(x-3)^3 \), the graph will also flatten out at \( x = 3 \).
Guess the equation of the following polynomial functions based on their graphs:

1. Write a polynomial equation, of smallest degree, for the function whose graph is shown below.

\[ y = a(x+6)(x+2)(x-3) \]

![Graph of (x+6) factor](image1)

![Graph of (x+2) factor](image2)

![Graph of (x-3) factor](image3)

Multiply these together to get the equation.

2. Write an equation of the polynomial of the smallest degree whose graph is shown below.

\[ y = a(x+2)^2(x-4) \]

Multiply these together to get the equation.

![Graph of (x+2) factor](image4)

![Graph of (x-4) factor](image5)

3. Find a formula for the polynomial function of smallest degree shown below. Only an algebraic solution will be accepted.

\[ y = a(x+5)(x+3)(x+5) \]

Multiply these together to get the equation.

![Graph of (x+5) factor](image6)

![Graph of (x+3) factor](image7)