Station A

A dairy farmer plans to fence in a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

1) \[ P_{\text{min}} = 2w + l \]

2) "Helper" \[ \frac{L \times W}{W} = \frac{180,000}{W} \]
   \[ L = \frac{180,000}{W} \]

3) Sub in \[ P_{\text{min}} = 2w + \frac{180,000}{w} \] or \[ 2w^2 + 180,000 \]

4) Find min/max \underline{GRAPHICALLY}

So if width = 300
then length = 600
(Total perimeter = 1200)
Station C

Determine the point(s) on $4x^2 + y^2 = 4$ that are farthest from $(1,0)$.

Optimum function

$$d_{\text{max}} = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

Helper

$4x^2 + y^2 = 4$ and $(1,0)$

$$y^2 = 4 - 4x^2$$

Sub in

$$d_{\text{max}} = \sqrt{(x-1)^2 + (y-0)^2}$$

$$= \sqrt{(x-1)^2 + y^2}$$

$$= \sqrt{(x-1)^2 + 4 - 4x^2}$$

$$= \sqrt{x^2 - 2x + 1 + 4 - 4x^2}$$

$$d_{\text{max}} = \sqrt{-3x^2 - 2x + 5}$$

$$x_{\text{max}} = \frac{-b}{2a} = \frac{-(-2)}{2(-4)} = \frac{2}{-8} = -\frac{1}{3}$$

So

$$y = \sqrt{4 - 4x^2}$$

$$y = \sqrt{4 - 4\left(-\frac{1}{3}\right)^2} = \pm \frac{2\sqrt{2}}{3}$$

and $$\left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$$

and $$\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$$
Station D

Let \( x \) and \( y \) be two positive numbers such that \( 2x + y = 30 \) and the product of \( (x+2) \) and \( y \) is a maximum.

Opt Func

\[
\text{Prod}_{\text{max}} = (x+2)(y)
\]

"Helper"

\[
2x + y = 30 \\
y = 30 - 2x
\]

Sub in

\[
\text{Prod}_{\text{max}} = (x+2)(30-2x)
\]

\[
\text{Prod}_{\text{max}} = -2x^2 + 26x + 60
\]

Fud Max Min

\[
X_{\text{max}} = \frac{-(-26)}{2(-2)} = \frac{-26}{-4} = \frac{13}{2} \text{ or } 6.5
\]

So \( y = 30 - 2x = 30 - 2(\frac{13}{2}) = 17 \)

\[
\begin{array}{c}
X = 6.5 \\
y = 17
\end{array}
\]
Station F

A rectangle is bounded by the x- and y-axes and the graph of \( y = 6 - 2x \) as shown below. Find the coordinate of the point of tangency where the rectangle will intersect the line so that the rectangle will have a maximum area.

\[
A_{\text{max}} = L \times W
\]

\[
A = (x)(y)
\]

"Help me"

\[ y = 6 - 2x \]

Sub in:

\[
A_{\text{max}} = (x)(6-2x)
\]

\[
A_{\text{max}} = -2x^2 + 6x
\]

Find \( x_{\text{max}} \):

\[
x = -\frac{b}{2a} = \frac{6}{2(-2)} = \frac{-6}{-4} = \frac{3}{2}
\]

\[
y = 6 - 2x = 6 - 2\left(\frac{3}{2}\right) = 3
\]
Station G

A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for $3 a foot, while the remaining two sides will use standard fencing selling for $2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of $6000?

\[ A_{\text{max}} = l \times w \]
\[ A_{\text{max}} = (x)(y) \]

\[ 6(2x) + 2(2y) = 6000 \]
\[ 6x + 4y = 6000 \]
\[ 6x \quad -6x \]
\[ 4y = 6000 - 6x \]
\[ y = 1500 - \frac{3}{2}x \]

Substitute:

\[ A_{\text{max}} = (x)\left(1500 - \frac{3}{2}x\right) \]
\[ A_{\text{max}} = -\frac{3}{2}x^2 + 1500x \]

Find Max/Min:

\[ x = \frac{-b}{2a} = \frac{-1500}{2(-\frac{3}{2})} = \frac{-1500}{-3} = 500 \]

Length = 500
Width = 750

\[ y = 1500 - \frac{3}{2}(500) = 750 \]
Station I  (Round to nearest $10^{45}$)

A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

\[ A_{\text{rec max}} = 2 \times w \]
\[ A_{\text{area max}} = 2 \times (y) \]

"Help!"

\[ y = 16 - x^2 \]

"Need help!"

\[ A_{\text{area max}} = 2 \times (16 - x^2) \]
\[ A_{\text{area max}} = -2x^3 + 32x \]

"Need help!"

Find Max/Min: Graphically!

2nd Trace Max: $x = 2.3$

\[ y = 16 - (2.3)^2 = 10.71 \]

\[ \text{so } 2x = 4.6 \]

and $y = 10.71$