Midterm Review Packet

**It is important to also study your previous review sheets, tests, and quizzes. This review packet should not be your only source of review or studying***
1. The distance that a person drives at a constant speed varies directly with the amount of time they have been driving. If, at a particular speed, a person drives 107 miles in two hours, then how far will they drive, at the same speed, in $1\frac{1}{4}$ hours?

(1) 75 miles  
(2) 91 miles  
(3) 44 miles  
(4) 67 miles

\[ \frac{\text{miles}}{\text{time}} = \frac{\text{miles}}{\text{time}} \]

\[ \frac{107}{2} = \frac{133.75}{x} \]

\[ x = 6.25 \]

2. Given the function $f(x)=x^2-2x+7$, what is its average rate of change over the interval $3 \leq x \leq 11$?

ROC = \frac{f(11) - f(3)}{11 - 3}

ROC = \frac{100 - 10}{11 - 3}

ROC = 12

3. At what $x$-coordinate would a line whose equation is $y=\theta x-3$ intersect a perpendicular line whose $y$-intercept is 17?

$(1) x = 12$  
$(2) x = -5$  
$(3) x = -11$  
$(4) x = 8$

$m = 2$  
$m = -\frac{1}{2}$  
$y = -\frac{1}{2}x + 17$

$2x - 3 = -\frac{1}{2}x + 17$

4. Selected values of a linear function $f(x)$ are given in the table below. Find the value of $k$. Explain how you found your answer.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-8$</th>
<th>$-2$</th>
<th>$4$</th>
<th>$12$</th>
<th>$14$</th>
<th>$18$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-33</td>
<td>-12</td>
<td>9</td>
<td>$k$</td>
<td>44</td>
<td>58</td>
</tr>
</tbody>
</table>

$m = \frac{-33 - 9}{-8} = \frac{12 - 4}{12 - 4}$

$m = \frac{7}{2}$ or $3.5$

$k - 9 = 37$

$k = \frac{49}{2}$

5. Write an equation for the line passing through the points $(-5, 15)$ and $(20, 25)$. Show how you arrived at your answer.

$L_1$  
$L_2$

$y = 0.4x + 17$

$a = 0.4$

$b = 17$

$\text{Stat} \quad \text{Calc}$

$y = ax + b$

4: LinReg
6. Solve the following system of equations algebraically.

\[
\begin{align*}
  5(3x - 5y + 2z) &= -5 \\
  -2(5x + y + 6z) &= 32
\end{align*}
\]

7. After a recent Arlington High School basketball game, traffic was exiting the parking lot at a constant rate of 28 cars per minute. The parking lot started with 922 cars.

a) How many cars are still in the parking lot after 10 minutes?

\[
28(10) = 280 \text{ cars have left}
\]

So \(922 - 280 = 642\) remain.

b) After 25 minutes, the rate at which the cars leave rises to 34 cars per minute. How many total minutes does it take for the parking lot to completely clear? Round to the nearest minute. Show your analysis.

\[
\frac{32}{25} \approx 1.28 \text{ minutes}
\]

The parking lot in the first 20 minutes, leaving 922 - 700 = 222 cars remaining.

\[
34m = 222 \Rightarrow m = \frac{222}{34} = 6.52941176
\]

c) Determine a formula for the number of cars, \(n\), in the parking lot after \(m\)-minutes.

\[
\begin{align*}
  n(m) &= 922 - 28m \\
  n(m) &= 922 - 28m \quad m \leq 25 \\
  n(m) &= 922 - 28(25) - 34m \quad m > 25
\end{align*}
\]
8. For the function \( f(x) = x^2 - 2x - 15 \), over which of the following intervals is \( f(x) > 0 \) always?

- (1) \( x > 0 \)
- (2) \( x < -5 \) or \( x > 3 \)
- (3) \( x < 0 \)
- (4) \( x < -3 \) or \( x > 5 \)

9. Selected values of a quadratic function are shown below.
   a) Which of the following values of \( x \) represents an \( x \)-intercept of the function? \( x \)-intercepts happen when \( y = 0 \). 
      \[ x = -6 \]
   b) Which of the following values of \( y \) represents a \( y \)-intercept of the function? \( y \)-intercepts happen when \( x = 0 \). 
      \[ y = -15 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-8</th>
<th>-6</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>0</td>
<td>-15</td>
<td>-16</td>
<td>-15</td>
</tr>
</tbody>
</table>

10. The function \( y = \frac{1}{2}(x - 0)^2 + 17 \) is strictly decreasing over what interval?

   \( x > 0 \)

11. The height of an object can be modeled by the equation \( h(t) = -16t^2 + 48t \).
   a) What is the initial height of the object?
      \( t = 0 \)
      \[ h(0) = -16(0)^2 + 48(0) \]
      \[ h(0) = 0 \]
   b) Will the object reach a maximum or minimum height? Explain.
      \[ \text{maximum} \]
   c) When will the object hit the ground?
      \[ -16t^2 + 48t = 0 \]
      \[ -16t(t - 3) = 0 \]
      \[ t = 3 \text{ sec.} \]
   d) Sketch the path of the object on the provided set of axes. Be sure to label!
12. Write the equations of the following parabolas using the given information.

- **a)** Vertex $(2, 8)$
  Directrix $y = 4$
  \[ y = \frac{1}{16}(x - 2)^2 + 8 \]

- **b)** Vertex $(0, 0)$
  Focus $(2, 0)$
  \[ x = \frac{1}{8}y^2 \]

- **c)** Focus $(0, 7)$
  Directrix $y = 3$
  \[ y = \frac{1}{8}x^2 + 5 \]

13. Identify the focus and the directrix of each parabola.

- **a)** \[ y = \frac{1}{2}x^2 \]
  Vertex $(0, 0)$
  Focus $(0, \frac{1}{4})$
  Directrix $y = -\frac{1}{2}$
  \[ \frac{1}{4}p = \frac{1}{2} \]
  \[ p = \frac{1}{2} \]

- **b)** \[ y = \frac{1}{4}x^2 \]
  Vertex $(0, 0)$
  Focus $(0, \frac{1}{2})$
  Directrix $y = 1$
  \[ \frac{1}{4}p = \frac{1}{2} \]
  \[ -4p = 4 \]
  \[ p = -1 \]

- **c)** \[ x^2 = -3y \]
  Vertex $(0, 0)$
  Focus $(0, \frac{3}{4})$
  Directrix $y = -\frac{3}{4}$
  \[ \frac{1}{4}p = \frac{3}{4} \]
  \[ -4p = -3 \]
  \[ p = \frac{3}{4} \]

- **d)** \[ y = \frac{1}{4}x^2 \]
  Vertex $(0, 0)$
  Focus $(0, \frac{1}{8})$
  Directrix $y = -\frac{1}{8}$
  \[ \frac{1}{4}p = \frac{1}{8} \]
  \[ 8p = 1 \]
  \[ p = \frac{1}{8} \]

- **e)** \[ x = \frac{1}{4}y^2 \]
  Vertex $(0, 0)$
  Focus $(\frac{1}{2}, 0)$
  Directrix $x = -\frac{1}{4}$
  \[ \frac{1}{4}p = \frac{1}{8} \]
  \[ -4p = 2 \]
  \[ p = -\frac{1}{2} \]

- **f)** \[ y = \frac{1}{36}x^2 \]
  Vertex $(0, 0)$
  Focus $(0, \frac{1}{36})$
  Directrix $y = 9$
  \[ \frac{1}{4p} = \frac{1}{36} \]
  \[ -4p = 36 \]
  \[ p = -9 \]
14. Using the graph at the right, it shows the height $h$ in feet of a small rocket $t$ seconds after it is launched. The path of the rocket is given by the equation: $h = -16t^2 + 128t$.

a) How long is the rocket in the air? $8$ sec.

b) What is the greatest height the rocket reaches? 256 ft

c) About how high is the rocket after 1 second? 112 ft

\[ h = -16(1)^2 + 128(1) \]

\[ h = 112 \]

d) After 2 seconds,

\[ h = -16(a)^2 + 128(a) \]

a. about how high is the rocket? 192 ft

b. is the rocket going up or going down? up

e) After 6 seconds,

\[ h = -16(a)^2 + 128(a) \]

a. about how high is the rocket? 192 ft

b. is the rocket going up or going down? down

f) Do you think the rocket is traveling faster from 0 to 1 second or from 3 to 4 seconds? Explain your answer.

\[ (0, 0) (1, 192) \]

\[ m = \frac{192}{1} = 192 \text{ ft/sec} \]

\[ (3, 240) (4, 256) \]

\[ m = \frac{256 - 240}{4 - 3} = 16 \text{ ft/sec} \]

\[ \frac{\text{Rate of change}}{h} = -16(4)^2 + 128(4) = 256 \]

\[ h = -16(3)^2 + 128(3) = 240 \]

g) Using the equation, find the exact value of the height of the rocket at 2 seconds.

\[ h = -16(a)^2 + 128(a) = 192 \text{ ft}. \]
Factor the following completely:

1. 

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>a)</td>
<td>$3x^3 - 75x$</td>
<td>b)</td>
</tr>
<tr>
<td></td>
<td>$3x(x^2 - 25)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3x(x+5)(x-5)$</td>
<td></td>
</tr>
</tbody>
</table>

| d) | $21x^4 + 35x^3 + 49x^2$ |
|    | $7x^2(3x^2 + 5x + 7)$ |

| e) | $24x^4 - 18x$ |
|    | $6x(4x^3 - 3)$ |

| f) | $15x^4 - 24x^3$ |
|    | $3x^2(5x^2 - 8)$ |

| g) | $a^2 + 3a + 4a + 12$ |
|    | $(a+3)(a+4)$ |

| h) | $y^2 - y + 5y - 5$ |
|    | $(y-1) + 5(y-1)$ |
|    | $(y-1)(y+5)$ |

| i) | $x^2 + 11x + 10$ |
|    | $(x+10)(x+1)$ |

| j) | $4x^2 + 12x - 72$ |
|    | $4(x^2 + 3x - 18)$ |

| k) | $8x^2 + 24x - 80$ |
|    | $8(x^2 + 3x - 10)$ |

| l) | $4x^2 - 16x - 84$ |
|    | $4(x^2 - 4x - 21)$ |
2. Solve the following equations by FACTORING!

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $5x(x-7)^2 (3x+2) = 0$</td>
<td>$x=0, x=7, x=-2/3$</td>
</tr>
<tr>
<td>b) $(x+4)^2 - 3(x+4) - 10 = 0$</td>
<td>$x=1, x=6$</td>
</tr>
<tr>
<td>c) $2(x^2 - 25) = 3x(25 - x^2)$</td>
<td>$x=5, x=15$</td>
</tr>
<tr>
<td>d) $x^2 + 3x - 4 = 50$</td>
<td>$x=-9, x=6$</td>
</tr>
<tr>
<td>e) $3x^2 - 8x + 4 = 0$</td>
<td>$x=2/3, x=2/13$</td>
</tr>
<tr>
<td>f) $3x^2 - 10x + 3 = 0$</td>
<td>$x=3/2, x=1/3$</td>
</tr>
</tbody>
</table>

3. Write the expression as a complex number in standard form.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $(5+2i) + (3-2i)$</td>
<td>$8$</td>
</tr>
<tr>
<td>b) $(-2+4i) + (3-9i)$</td>
<td>$1-5i$</td>
</tr>
<tr>
<td>c) $3i(6-5i)$</td>
<td>$18i - 15i^2 = 18i + 15$</td>
</tr>
<tr>
<td>d) $(5-2i) - 2(3+i)$</td>
<td>$5-2i - 6-2i = -1-4i$</td>
</tr>
<tr>
<td>e) $i(2+i)$</td>
<td>$2i + i^2 = 1-1+2i$</td>
</tr>
<tr>
<td>f) $(2+3i)(1-4i)$</td>
<td>$2-8i + 3i - 12i^2 = 2 - 5i - 12(-1) = 14 - 5i$</td>
</tr>
<tr>
<td>g) $(-3+7i)(1-2i)$</td>
<td>$-3+6i + 7i - 14i^2 = -3 + 13i + 14(-1) = 11 + 13i$</td>
</tr>
<tr>
<td>h) $(3-2i)^2$</td>
<td>$(3-2i)(3-2i) = 9 - 6i - 6i + 4i^2 = 9 - 12i + 4(-1) = 5 - 12i$</td>
</tr>
</tbody>
</table>
4. Simplify each radical expression.

<table>
<thead>
<tr>
<th>a) (- \frac{2}{3} \sqrt{-9} )</th>
<th>b) (\frac{3}{4} \sqrt{-144} )</th>
<th>c) (\frac{3}{5} \sqrt{-\frac{100}{9}} )</th>
<th>d) (\sqrt{-\frac{1}{4}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{-2}{3}(3i)) (\sqrt{-2i})</td>
<td>(\frac{3}{4}(12i)) (\sqrt{9i})</td>
<td>(\frac{3}{5}(\frac{10i}{3})) (\sqrt{\frac{10}{9}})</td>
<td>(\sqrt{-\frac{1}{4}} ) (\frac{1}{2})</td>
</tr>
<tr>
<td>e) (2\sqrt{-75}) (\frac{2(5i\sqrt{3})}{10i\sqrt{3}})</td>
<td>f) (3\sqrt{-11}) (\sqrt{3i\sqrt{11}})</td>
<td>g) (-\sqrt{-10}) (-i\sqrt{10})</td>
<td>h) (-\frac{1}{2}\sqrt{-300}) (-\frac{1}{2}(10i\sqrt{3})) (-5i\sqrt{3})</td>
</tr>
</tbody>
</table>

5. Simplify:

<table>
<thead>
<tr>
<th>a) (i^{12}) (\frac{12}{4} = 3.0)</th>
<th>b) (i^{99}) (\frac{99}{4} = 24.75)</th>
<th>c) (i^{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= \sqrt{1})</td>
<td>(= \sqrt{-1})</td>
<td>(= \sqrt{-1})</td>
</tr>
<tr>
<td>d) (2i^{2} \cdot (3i)^{3}) (2i^{2}(-27i) = -54i^{3})</td>
<td>e) (i^{8} + i^{9} + i^{10}) (= 1i)</td>
<td>f) (i^{8} \cdot i^{9} \cdot i^{10}) (\frac{27}{4} = 6.75)</td>
</tr>
<tr>
<td>(= i\sqrt{54}) (-54(-i))</td>
<td>(= 1i) (= 1i)</td>
<td>(i^{27} = i)</td>
</tr>
</tbody>
</table>
6. Solve by using the Quadratic Formula. Express your answer in simplest radical form or simplest \(a + bi\) form as necessary.

\[
(x+5)(x+5) = x^2 + 10x + 25
\]
\[
x^2 + 10x + 25 + 10 = 2
\]
\[
x^2 + 10x + 33 = 0
\]
\[
a = 1, \quad b = 10, \quad c = 33
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-10 \pm \sqrt{100 - 4(1)(33)}}{2(1)}
\]
\[
x = \frac{-10 \pm \sqrt{100 - 132}}{2}
\]
\[
x = \frac{-10 \pm \sqrt{-32}}{2}
\]
\[
x = \frac{-5 \pm 2\sqrt{8}}{1}
\]
\[
x = -5 \pm 2i\sqrt{2}
\]

\[
6x^2 + 1 = -5
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-1 \pm \sqrt{1 + 120}}{12}
\]
\[
x = 0 \pm 2\sqrt{15}
\]

\[
4x^2 - 20 = 0
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{0 \pm \sqrt{0 - 4(-16)}}{8}
\]
\[
x = 0 \pm \sqrt{5}
\]

7. Solve by using the completing the square method. Express your answer in simplest radical form or simplest \(a + bi\) form as necessary.

\[
4x^2 - 24x + 28 = 0
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{6 \pm \sqrt{36 - 4(4)(7)}}{8}
\]
\[
x = \frac{6 \pm \sqrt{36 - 112}}{8}
\]
\[
x = \frac{6 \pm \sqrt{-72}}{8}
\]
\[
x = \frac{3 \pm \sqrt{18}}{4}
\]
\[
x = \frac{3 \pm 3\sqrt{2}}{4}
\]

\[
x^2 + 2x + 5 = 0
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2}
\]
\[
x = \frac{-2 \pm \sqrt{-16}}{2}
\]
\[
x = \frac{-2 \pm 4i}{2}
\]
\[
x = -1 \pm 2i
\]

\[
3x^2 - 6x - 12 = 0
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{6 \pm \sqrt{36 - 4(3)(-12)}}{6}
\]
\[
x = \frac{6 \pm \sqrt{36 + 144}}{6}
\]
\[
x = \frac{6 \pm \sqrt{180}}{6}
\]
\[
x = \frac{6 \pm 6\sqrt{5}}{6}
\]
\[
x = 1 \pm \sqrt{5}
\]

\[
x^2 - 6x + 9 = -2 + 9
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{6 \pm \sqrt{36 - 4(3)(-3)}}{6}
\]
\[
x = \frac{6 \pm \sqrt{36 + 36}}{6}
\]
\[
x = \frac{6 \pm \sqrt{72}}{6}
\]
\[
x = \frac{6 \pm 6\sqrt{2}}{6}
\]
\[
x = 1 \pm \sqrt{2}
\]
8. Convert each quadratic equation to vertex form by completing the square.

a) \[ y = x^2 + 26x + 50 \]
\[ y - 119 = (x + 13)^2 - 119 \]
\[ y = (x + 13)^2 - 119 \]

b) \[ y = -2x^2 + 20x - 10 \]
\[ \frac{y + 10}{2} = x^2 - 10x + 5 \]
\[ \frac{y + 10}{2} = (x - 5)^2 - 25 \]
\[ y = -(x - 5)^2 + 25 \]

9. Solve the following system of equations algebraically.

a) \[ y = x^2 + 5x - 10 \]
\[ y - 18 = 2x \]
\[ x^2 + 5x - 10 = 2x + 18 \]
\[ -2x \]
\[ -18 \]
\[ -2x - 18 \]
\[ x^2 + 3x - 28 = 0 \]
\[ x^2 + 7x - 4x - 28 \]
\[ x(x+7) - 4(x+7) = 0 \]
\[ (x+7)(x-4) = 0 \]
\[ x = -7 \]
\[ x = 4 \]
\[ y = 2(-7) + 18 \]
\[ y = 2\cdot4 + 18 \]
\[ y = 26 \]
\[ (7, 4) \]
\[ (4, 26) \]

b) \[ x^2 + y^2 = 100 \]
\[ y - x = 2 \]
\[ x = y + 2 \]
\[ x^2 + (y + 2)^2 = 100 \]
\[ x^2 + x^2 + 4x + 4 = 100 \]
\[ 2x^2 + 4x + 4 = 100 \]
\[ 2x^2 + 4x - 96 = 0 \]
\[ 2(x^2 + 2x - 48) = 0 \]
\[ x = -8 \]
\[ x = 8 \]
\[ (-8, -6) \]
\[ (-8, 6) \]
\[ \sqrt{2} \]
10. Solve each inequality and express the solution set in interval notation.

a) \( x - x^2 > 0 \)
\[
\frac{-x^2 + x > 0}{-1}
\]
\[
x - x \leq 0
\]
\[
(x-1) \leq 0
\]
\[
0 \leq x \leq 1
\]

b) \( x^2 - 6x + 5 < 0 \)
\[
\frac{x^2 - 5x - 1x + 5}{m:5}
\]
\[
(x-5) - 1(x-5) < 0
\]
\[
(x-5)(x-1) < 0
\]
\[
1 \leq x \leq 5
\]

11. Solve each inequality graphically.

a) \( y + 1 \geq x^2 + 4x \)
\[
y \geq x^2 + 4x - 1
\]

b) \( y < x^2 - 4 \)
\[
\frac{x}{3} \leq 0
\]
\[
\frac{y}{5} \leq 0
\]
12. Perform the following polynomial operations.

\[\text{a) } (5 + 7x + 3x^2) + (7x^2 + 9 - 7x)\]

\[= 10x^2 + 14\]

\[\text{b) } (4y^3 - 6y + 8y^2) - (-3y^2 - 7 + 2y^3)\]

\[= 2y^3 + 11y^2 - 6y + 7\]

\[\text{c) } (x + 4)(6x^2 + 2x - 8)\]

\[\begin{array}{c|ccc}
 x & 6x^3 & 2x^2 & -8y \\
+4 & 24x^2 & 8x & -32 \\
\hline
= 60x^3 + 26x^2 - 32
\end{array}\]

\[\text{d) } (n + 4m)(2n - 3m)\]

\[= 2n^2 - 3mn + 8mn - 12m^2\]

\[= 2n^2 + 5mn - 12m^2\]

\[\text{e) } 7rs(4r^2 + 9s^2 - 7rs)\]

\[= 28r^3 s + 63rs^4 - 49r^2 s^6\]

\[\text{f) } (-3x^3 + 5xy - 2y^2) - (y^2 + 5xy - 9y)\]

\[-3x^3 + 5xy - 2y^2 - y^2 - 5xy + 9y\]

\[-3x^2 - 3y^2 + 9y\]

\[\text{g) Find the perimeter of the triangle pictured.}\]

\[P = 14x^2 + 19y + 4\]

\[\text{h) Michelle borrowed } 3r^3 + 5r^2 + 18r + 20 \text{ dollars from her brother. If she paid back } 3r^3 + 2r^2 - 2r + 11 \text{ dollars, then how much more money does she still owe her brother?}\]

\[\begin{array}{c}
(3r^3 + 5r^2 + 18r + 20) - (3r^3 + 2r^2 - 2r + 11) \\
= 3r^3 + 5r^2 + 18r + 20 - 3r^3 - 2r^2 + 2r - 11 \\
= 3r^2 + 20r + 9
\end{array}\]
13. Divide each polynomial by the method indicated. (L)-long (S)-synthetic (RT)-Reverse Tabular

a) \((x^3 + 13x^2 + 39x + 20) \div (x + 9)\) \((S)\)  
\[\begin{array}{c|cccc}
1 & 13 & 39 & 20 & \text{x+9}=0 \\
-9 & & -9 & -36 & -27 \\
\hline
1 & 4 & 3 & -7 \\
\end{array}\]
\[x^2 + 4x + 3 + \frac{-7}{x+9}\]

b) \((6t^4 + 4t^3 - 13t^2 - 10t - 5) \div (2t - 5)\) \((L)\)
\[\begin{array}{c|ccccccc}
2t^2 - 5 & 3t^2 + 2t + 1 & \text{x=-9} \\
6t^4 & & & & & & & & \\
-15t^2 & & & & & & & & \\
-4t^3 & & & & & & & & \\
-2t^2 & & & & & & & & \\
-10t & & & & & & & & \\
-5 & & & & & & & & \\
\hline
3t^2 + 2t + 1 & & & & & & & & \\
0 & & & & & & & & \\
\end{array}\]

c) \((x^3 + 5x^2 - 2x - 24) \div (x^2 + 7x + 12)\) \((RT)\)
\[\begin{array}{c|cc}
2 & \text{x} & \text{-2} \\
7x & \text{x^3} & \text{-2x^2} \\
12 & \text{7x^2} & \text{-14x} \\
+2 & \text{12x} & \text{-24} \\
\hline
\text{x^3} & \text{5x^2} & \text{2x} - 24 \\
\end{array}\]
\[= (x-2)\]

d) \((4n^3 - 13n - 6) \div (2n + 1)\) \((RT)\)
\[\begin{array}{c|cc}
2n^2 - n & \text{2n^2} & \text{-n} - 6 \\
4n^3 & \text{2n^2} & \text{-n} - 6 \\
+1 & \text{4n^3} & \text{0n^2} - 13n - 6 \\
\hline
2n^2 & \text{4n^3} & \text{-2n^2} - 13n - 6 \\
-2n^2 & \text{-2n^2} & \text{2n^2} - 13n - 6 \\
-2n^2 & \text{-2n^2} & \text{2n^2} - 13n - 6 \\
\hline
0 & \text{-12n} & \text{+6} \\
\end{array}\]

e) \((x^3 - 10x - 15) \div (x + 8)\) \((S)\)  
\[\begin{array}{c|cccc}
1 & 7 & -10 & -15 & \text{x+8}=0 \\
1 & -8 & 8 & 16 \\
\hline
1 & -1 & -2 & 1 \\
\end{array}\]
\[x^2 - 1x - 2 + \frac{1}{x+8}\]

f) \((10a^4 - a^3 + 11a^2 + 7a + 5) \div (5a^2 + 2a - 1)\) \((L)\)
\[\begin{array}{c|ccccccc}
2a^2 - a + 3 & 10a^4 & -a^3 & 11a^2 & 7a & 5 & \text{5a^2 + 2a - 1} \\
& 2a & -a^3 & 11a^2 & 7a & 5 & 2a & -1 \\
& -5a^3 & +13a^2 & 7a & 5 & 2a & -1 \\
& 5a^3 & -2a^2 & -a \\
\hline
2a^2 - a + 3 & \text{15a^2} & +10a & 5 & \text{-15a^2 + 10a + 3} \\
\end{array}\]
\[= \frac{2a^2 - a + 3 + 8}{5a^2 + 2a - 1}\]
1. Expand the following expressions using Pascal’s Triangle.

a) \((4n+3)^3\)

\[
\begin{align*}
1(4n)^0(3)^3 + 3(4n)^2(3)^1 + 3(4n)(3)^2 + 1(3)^3 \\
1(64n^3)(1) + 3(16n^2)(3) + 3(4n)(9) + 1(1)(27) \\
\frac{64n^3 + 144n^2 + 108n + 27}{64n^3 + 144n^2 + 108n + 27}
\end{align*}
\]

b) \((2a-2)^4\)

\[
\begin{align*}
1(2a)^0(-2)^4 + 4(2a)^3(-2)^1 + 6(2a)^2(-2)^2 + 4(2a)(-2)^3 + 1(-2)^4 \\
1(16a^4)(1) + 4(8a^3)(-2) + 6(4a^2)(4) + 4(2a)(-8) + 1(1)(16) \\
\frac{16a^4 - 64a^3 + 144a^2 - 64a + 16}{16a^4 - 64a^3 + 144a^2 - 64a + 16}
\end{align*}
\]

c) \((b+4)^5\)

\[
\begin{align*}
1(b)^0(-4)^5 + 5(b)^4(-4)^1 + 10(b)^2(-4)^2 + 5(b)^1(-4)^3 + 1(-4)^4 \\
1(b^5)(1) + 5(b^4)(4) + 10(b^3)(16) + 5(b^2)(64) + 1(b)(256) + 1(1)(1024) \\
b^5 + 20b^4 + 160b^3 + 640b^2 + 572b + 1024
\end{align*}
\]

2. Find the specific term in each expansion.

\[\text{4}^{\text{th}} \text{ Term}\]

a) 4\text{th} term of \((n+3)^6\)

\[
\begin{align*}
1(6)^0(3)^3 &= 20 \frac{n^3}{n^3(27)} = \frac{540 n^3}{n^3}
\end{align*}
\]

b) 5\text{th} term of \((2x-3y)^7\)

\[
\begin{align*}
35(2x)^3(-3y)^4 &= 35(8x^3)(81y^4) = 2,268,000 x^3 y^4
\end{align*}
\]

c) 3\text{rd} term of \(\left(\frac{a-1}{a}\right)^3\)

\[
\begin{align*}
10(a)^0 \left(-\frac{1}{a}\right)^2 &= 10(a^3) \left(-\frac{1}{a^2}\right) = 10a
\end{align*}
\]

d) 2\text{nd} term of \((3-x^2)^4\)

\[
\begin{align*}
4(3)^3(-x^2)^2 &= 4(27)(-x^2) = -108 x^2
\end{align*}
\]
3. Use synthetic division to evaluate each function at the specified value of x.

(a) \( f(x) = 20x^4 - 8x^3 + 25x^2 + 50x - 16 \) for \( f(-5) \).

\[
\begin{array}{c|cccc}
-5 & 20 & -8 & 25 & 50 & -16 \\
  &  & -100 & 590 & -2825 & 13875 \\
\hline
  & 20 & -108 & 565 & -2775 & 13859 \\
\end{array}
\]

\( f(-5) = 13859 \)

(b) Find \( h(-2.3) \) using synthetic substitution

\( h(x) = 2x^3 + 3x^2 - 7x + 1 \)

\[
\begin{array}{c|cccc}
-2.3 & 2 & 3 & -7 & 1 \\
  & 2 & -1.6 & -3.32 & 8.636 \\
\hline
\end{array}
\]

\( h(-2.3) = 8.636 \)

c) \( P(x) = 4x^3 - 12x^2 - 2 \) for \( x = 5 \)

\[
\begin{array}{c|cccc}
5 & 4 & 0 & -12 & -2 \\
  & 20 & 100 & 440 \\
\hline
  & 4 & 20 & 88 & 438 \\
\end{array}
\]

\( P(5) = 438 \)

d) \( P(x) = -3x^4 + 5x^3 - x + 7 \) for \( x = -2 \)

\[
\begin{array}{c|cccc}
-2 & -3 & 5 & 0 & -1 & 7 \\
  & 6 & -22 & 44 & -86 \\
\hline
  & -3 & 11 & -22 & 43 & -79 \\
\end{array}
\]

\( P(-2) = -79 \)

4. Solve the following polynomial equations (factorable).

(a) \( x^3 - x^2 - 2x = 0 \)

\[
\begin{align*}
X(x^2 - x - 2) &= 0 \\
X(x-2)(x+1) &= 0
\end{align*}
\]

\( \begin{array}{c|c|c}
X = 0 & X = 2 & X = -1 \\
\end{array} \)

(b) \( 0 = -x^3 + 2x^2 + 4x - 8 \)

\[
\begin{align*}
x^3 - 2x^2 - 4x + 8 &= 0 \\
x^2(x-2) - 4(x-2) &= 0 \\
(x-2)(x^2-4) &= 0 \\
(x-2)(x+2)(x-2) &= 0
\end{align*}
\]

\( \begin{array}{c|c|c}
X = 2 & X = -2 & X = 2 \\
\end{array} \)

c) \( x^3 - 3x^2 + 9x - 27 = 0 \)

\[
\begin{align*}
x^2(x-3) + 9(x-3) &= 0 \\
(x-3)(x^2+9) &= 0
\end{align*}
\]

\( \begin{array}{c|c}
X = 3 & x^2+9 = 0 \\
  & x^2 = -9 \\
  & X = \pm 3i \\
\end{array} \)

(d) \( x^3 - 5x^2 + 5x - 25 = 0 \)

\[
\begin{align*}
x^2(x-5) + 5(x-5) &= 0 \\
(x-5)(x^2+5) &= 0
\end{align*}
\]

\( \begin{array}{c|c}
X = 5 & x^2+5 = 0 \\
  & x^2 = -5 \\
  & X = \pm \sqrt{-5} = \pm \sqrt{5}i \\
\end{array} \)
5. Solve the following polynomial equations. Find all zeros (real and imaginary). (unfactorable)

a) \( x^3 + 7x^2 - 5x - 18x = 0 \)

\[
\begin{align*}
X^3 & - 5X^2 - 18X + 72 = 0 \\
\text{Find an integer root on the graph} & \\
\text{Look on table!} & \\
X = 3 & \to Y = 0
\end{align*}
\]

\[
\begin{array}{c|ccc|c}
X & 1 & -6 & -18 & 72 \\
--- & -1 & 6 & -24 & 0 \\
\hline
1 & -2 & -24 & 0 & 0
\end{array}
\]

\[
X^2 - 2X - 24 = 0 \\
(X - 6)(X + 4) = 0
\]

\[
\begin{align*}
X &= 6 \\
X &= -4
\end{align*}
\]

b) \( x^4 - 6x^3 + 10x^2 - 6x + 9 = 0 \)

\[
\begin{align*}
\text{Look on graph!} & \\
X = 3 \text{ is double root} & \\
(\text{use it twice!})
\end{align*}
\]

\[
\begin{array}{c|cccc|c}
3 & 1 & -6 & 10 & -6 & 9 \\
-3 & 3 & -9 & 3 & -9 \\
\hline
1 & -3 & 1 & -3 & 0 \\
\hline
0 & 1 & 0 & 0 & 0
\end{array}
\]

\[
\begin{align*}
X^2 + 1 &= 0 \\
X &= \pm i
\end{align*}
\]

So \( X = 3, 3, -i, -i \)


c) \( 3x^3 + 12x^2 + 3x - 18 = 0 \)

\[
\begin{align*}
\text{Use graph to find} X &= 1 \to Y = 0 \\
3 & \mid 1 & 4 & 1 & -6 \\
1 & 5 & 6 & 0 \\
\hline
1 & 5 & 6 & 0
\end{align*}
\]

\[
X^2 - 2x + 6 = 0 \\
(X + 3)(X + 2) = 0
\]

\[
\begin{align*}
X &= -3 \\
X &= -2
\end{align*}
\]

d) \( 3x^2 + x^4 - 24x - 81 = 0 \)

\[
\begin{align*}
\text{Use graph to find} & \\
X &= -3 \to Y = 0 \\
-3 & \mid 3 & 1 & 0 & 0 & -24 & -81 \\
9 & 24 & -72 & 216 & 81 \\
\hline
3 & -8 & 24 & -72 & -27 & 0 \\
\hline
2 & 9 & 3 & 81 & 27 & 0
\end{align*}
\]

\[
\begin{align*}
X &= \frac{1}{3} \to Y = 0 \\
3 & \mid 1 & 27 & 9 \\
\hline
\frac{1}{3} & 3 & 1 & 27 & 9 \\
\hline
0 & 27 & 0 & 0
\end{align*}
\]

\[
\begin{align*}
3x^2 + 27 &= 0 \\
X &= \frac{-9}{3} \\
X &= \pm 3i
\end{align*}
\]

So .....

\[
\begin{align*}
X &= -3, 3 \\
X &= \frac{1}{3}, \pm 3i
\end{align*}
\]
6. Fill in the blanks for the end behavior for each of the following functions.

<table>
<thead>
<tr>
<th></th>
<th>f(x) = x^3 - 5x</th>
<th>f(x) = -x^3 - 3x^2 + 2</th>
<th>f(x) = x^4 - 4x^2 + x</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>f(x) \rightarrow -\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow -\infty</td>
</tr>
<tr>
<td></td>
<td>and</td>
<td>and</td>
<td>and</td>
</tr>
<tr>
<td></td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow -\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow -\infty as x \rightarrow +\infty</td>
</tr>
<tr>
<td>d)</td>
<td>f(x) = x + 12</td>
<td>f(x) = -x^2 + 3x + 1</td>
<td>f(x) = -x^8 + 9x^4 - 2x^4</td>
</tr>
<tr>
<td></td>
<td>f(x) \rightarrow -\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow -\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow -\infty as x \rightarrow -\infty</td>
</tr>
<tr>
<td></td>
<td>and</td>
<td>and</td>
<td>and</td>
</tr>
<tr>
<td></td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
</tr>
<tr>
<td>g)</td>
<td>f(x) = 4x^6 - 3x^3 + 5x - 2</td>
<td>f(x) = -2x^3 + 5x^2</td>
<td>f(x) = 6x^3 + 1</td>
</tr>
<tr>
<td></td>
<td>f(x) \rightarrow +\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow -\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow -\infty</td>
</tr>
<tr>
<td></td>
<td>and</td>
<td>and</td>
<td>and</td>
</tr>
<tr>
<td></td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
<td>f(x) \rightarrow +\infty as x \rightarrow +\infty</td>
</tr>
</tbody>
</table>

7. A 4\textsuperscript{th} degree polynomial with real coefficients is found to have exactly two distinct real roots. What must be true about the other two roots?

1. One root is real and the other is imaginary.
2. Both roots must be real.
3. Both roots are complex conjugates.
4. All the roots have been found.

8. Given polynomial \( q(x) \), \( q(4) = 6 \). Which statement is correct?

1. \( x = 4 \) is not a root
2. \( x = 4 \) is a root
3. \( (x - 4) \) is a factor
4. \( (x + 4) \) is not a factor
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a) $f(x) = x^3 - 3x^2$
  \[x = 0 \quad x = 3\] | b) $f(x) = -2(x+3)^2(x+1)^2$
  \[x = -3 \quad x = -1\] | c) $f(x) = -(x-1)(x-3)(x-5)$
  \[x = 1 \quad x = 3 \quad x = 5\] |
| d) $f(x) = -2x^3 + 8x$
  \[x = -2 \quad x = 2\] | e) $f(x) = -x^3 + 9x$
  \[x = 3\] | f) $f(x) = -2x^2 + 16x - 24$ |
| g) $f(x) = x$ | h) $f(x) = 3x^4 - 3x^3 - 3x^2 + 3x$ | i) $f(x) = -5$ |
| j) $f(x) = -3(x-1)(x-2)(x-3)$
  \[x = 1 \quad x = 2 \quad x = 3\] | k) $f(x) = x^4 - 3x^3$
  \[x = 0 \quad x = 3\] | l) $f(x) = x^2(x-3)$
  \[x = 0 \quad x = 3\] |
| m) $f(x) = 4x^2 - 9$
  \[x = -3 \quad x = 3\] | n) $f(x) = x^2(x-3)^3$
  \[x = 0 \quad x = 3\] | o) $f(x) = -(x-4)(x-3)(x-1)^2$
  \[x = 1 \quad x = 3 \quad x = 5\] |
10. Write a possible equation, in factored form, for a polynomial.

\[ f(x) = a(x-5)^3(x+2)^2(x-3) \]

Can be any \( a \neq 0 \! \)

\[ h(x) = \text{\( a \) has a degree of 6 with a positive leading coefficient and zeroes of 0, -5 with a multiplicity of 3, and \( \frac{3}{2} \) with a multiplicity of 1.} \]

11. Simplify:

a) \( \frac{\sqrt{32}}{\sqrt{4}} = \sqrt{8} = \sqrt{2^3} = 2\sqrt{2} \)

b) \( \frac{\sqrt{162x^5}}{\sqrt{3x^3}} = \frac{3\sqrt{2}x^2}{\sqrt{3}x} = \sqrt{2}x \)

c) \( \frac{\sqrt{1024x^{11}}}{\sqrt{4}x} = \sqrt{256x^{10}} = 16x^5 \)

d) \( \sqrt{2p^2q^2} = \sqrt{2}pq \)

e) \( \sqrt{25x^4} = 5x^2 \)

f) \( \sqrt{45a^2c^3} = 3ac\sqrt{5c} \)

12. Simplify:

a) \( (14a^ib^j)(a^c) = 14a^{i+c}b^j \)

b) \( (4h^j)(-2g^k) = -8h^jg^k \)

c) \( (3y^2z)^2(x^2y^2z) = 9y^4z^2x^2y^2z \)

d) \( \frac{-2d^{11}f^{18}}{c^{36}} \)

\( (-2d^{11}f^{18}c^{-18})^2 = \frac{4d^{22}f^{36}}{c^{36}} \)
13. Graph the following radical functions:

a) \( f(x) = \sqrt{x-2} + 4 \)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2.41</td>
</tr>
<tr>
<td>-1</td>
<td>2.56</td>
</tr>
<tr>
<td>0</td>
<td>2.74</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

b) \( f(x) = \sqrt{3x-6} \)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR</td>
<td>ERROR</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>ERROR</td>
</tr>
<tr>
<td>2</td>
<td>1.73</td>
</tr>
<tr>
<td>3</td>
<td>2.45</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

C) \( y = \sqrt{x-2} + 5 \)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ERROR</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6.41</td>
</tr>
<tr>
<td>5</td>
<td>6.73</td>
</tr>
</tbody>
</table>

D) \( y = -\sqrt{x+8} + 3 \)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR</td>
<td>ERROR</td>
</tr>
<tr>
<td>-9</td>
<td>ERROR</td>
</tr>
<tr>
<td>-8</td>
<td>ERROR</td>
</tr>
<tr>
<td>-7</td>
<td>2</td>
</tr>
<tr>
<td>-6</td>
<td>1.59</td>
</tr>
</tbody>
</table>

22
14. Solve the following equations. Check your answers!

a) \[ \frac{2 + \sqrt{3x - 2}}{-2} = \frac{-a}{3} \]
\[ \left( \sqrt{3x - 2} \right)^2 = \left( \frac{4}{3} \right)^2 \]
\[ 3x - 2 = 16 \]
\[ + \frac{a}{3} = +a \]
\[ 8x = 18 \]
\[ x = 3 \]
\[ x = 6 \checkmark \]

b) \[ \left( \frac{x}{3} \right)^3 = \left( \frac{-8}{3} \right)^3 \]
\[ x = -\frac{512}{27} \]

\[ \begin{array}{c}
1.\ \sum 2x - 1 = 10 \\
\quad -7\quad -7 \\
\quad \frac{(3x - 1)^3}{3} = (3)^3 \\
\quad 2x - 1 = 27 \\
\quad + (\quad +1) \\
\quad 2x = 28 \\
\quad x = 14 \\
\quad \sqrt{19} \checkmark \\
\end{array} \]

c) \[ \sqrt{5x + 1} + 5 = 3x \]
\[ \left( \sqrt{5x + 1} \right)^2 = (3x - 5)^2 \]
\[ 5x + 1 = 9x^2 - 30x + 25 \]
\[ -5x = -9x^2 + 30x - 24 \]
\[ 0 = 9x^2 - 35x + 24 \]
\[ (9x - 8)(x - 3) = 0 \]
\[ x = \frac{8}{9} \quad \text{and} \quad x = 3 \]

\[ \begin{array}{c}
\text{d) } 5x + 1 = 3x + 5 \\
\quad \frac{x - 5}{-5} = \frac{-5}{5} \\
\quad (\sqrt{x + 1})^2 = (3x - 5)^2 \\
\quad 5x + 1 = 9x^2 - 30x + 25 \\
\quad -5x = 9x^2 - 35x + 24 \]
\[ 0 = 9x^2 - 35x + 24 \]
\[ (9x - 8)(x - 3) = 0 \]
\[ x = 8 \text{ and } x = 3 \]
\[ \checkmark \text{ and } \checkmark \]

\[ \begin{array}{c}
\text{e) } 3x^2 - 2 = x^2 + 4x + 4 \\
\quad \frac{-2}{x^2 - 4x - 4} \\
\quad 2x^2 - 4x - 6 = 0 \quad 2 \\
\quad \frac{x^2 - 2x - 3 = 0}{x = 3, -1} \quad \checkmark \quad \checkmark \\
\quad (x - 3)(x + 1) = 0 \]
\[ x = 3 \quad \checkmark \quad x = -1 \quad \checkmark \]

\[ \begin{array}{c}
f) \left( \frac{x^2 - 1}{2} \right)^3 = \left( \frac{2}{3} \right)^3 \\
\quad x^2 - 1 = 8 \\
\quad + \frac{x}{2} = -\frac{8}{2} \\
\quad x^2 - 9 = 0 \\
\quad (x + 3)(x - 3) = 0 \\
\quad x = -3 \quad \text{and} \quad x = 3 \quad \checkmark \quad \checkmark \\
\end{array} \]
15. Use the functions below to answer the given questions:

\[ f(x) = 3x - 4 \quad g(x) = 2x^2 + 5 \quad h(x) = 8 - 3x \quad p(x) = x^2 - 2x \]

<table>
<thead>
<tr>
<th>a) ( (h + g)(3) )</th>
<th>b) ( (f - p)(-1) )</th>
<th>c) ( (h \cdot p)(5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{(3x-4) + (2x^2 + 5)}{6 - 4 + 18 + 5} )</td>
<td>( \frac{(3x-4) - (x^2 - 2x)}{(-7) - (3)} )</td>
<td>( \frac{(-3x)(x^2 - 2x)}{(-7)(15)} )</td>
</tr>
<tr>
<td>15</td>
<td>-10</td>
<td>-105</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d) ( (f \cdot g)(2) )</th>
<th>e) ( (f - h)(x) )</th>
<th>f) ( (p + g)(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{(3x-4)(2x^2 + 5)}{(2)(13)} )</td>
<td>( \frac{(3x-4) - (8 - 3x)}{6x - 12} )</td>
<td>( \frac{x^2 + 2x}{2x^4 + 5x^2 - 4x^3 - 10x} )</td>
</tr>
<tr>
<td>26</td>
<td>6x - 12</td>
<td>2x^4 - 4x^3 + 5x^2 - 10x</td>
</tr>
</tbody>
</table>

16. Find the inverse (if it exists) for each function.

<table>
<thead>
<tr>
<th>a) ( f(x) = x + 4 )</th>
<th>b) ( f(x) = \sqrt{x} )</th>
<th>c) ( f(x) = 3x^2 - 6x + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x + 4 )</td>
<td>( y = \frac{\sqrt{x}}{3} )</td>
<td>( y = \frac{x^3}{3} )</td>
</tr>
<tr>
<td>( x = y + 4 )</td>
<td>( x = (3y)^{\frac{1}{3}} )</td>
<td>( y = x^3 )</td>
</tr>
<tr>
<td>( y = x - 4 )</td>
<td>( y = \sqrt{x} )</td>
<td>( f^{-1}(x) = x^3 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d) ( f(x) = x^3 - 1 )</th>
<th>e) ( y = \frac{x + 4}{x + 3} )</th>
<th>f) ( y = \frac{x - 3}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^3 - 1 )</td>
<td>( x = \frac{y + 4}{y + 3} )</td>
<td>( x = \frac{y - 3}{2} )</td>
</tr>
<tr>
<td>( x = y^3 - 1 )</td>
<td>( xy + 3x = y + 4 )</td>
<td>( 2x = y - \frac{3}{2} )</td>
</tr>
<tr>
<td>( y = \sqrt[3]{x + 1} )</td>
<td>( x + 3 = y - \frac{3}{2} )</td>
<td>( + 3 = \frac{y + 3}{2} )</td>
</tr>
<tr>
<td>( y = \sqrt[3]{x + 1} )</td>
<td>( x - 1 = 4 - 3x )</td>
<td>( y^{-1} = 2x + 3 )</td>
</tr>
</tbody>
</table>

Fails the horizontal test so no inverse!
1. You start an account with $500 and an interest rate of 6% compounded yearly. How much is in the account after 3 years?

\[ V = 500 \times (1 + 0.06)^3 \]
\[ V = 595.508 \]
\[ V \approx 595.51 \]

2. From 2000 - 2010 a city had a 2.5% annual decrease in population. If the city had 2,950,000 people in 2000, determine the city's population in 2008.

\[ V = 2950000 \times (1 - 0.025)^8 \]
\[ V = 2409122.821 \]
\[ V \approx 2409123 \]

3. You buy a car for $8000 that depreciates at a rate of 11% a year. How much is the car worth after 5 years?

\[ V = 8000 \times (1 - 0.11)^5 \]
\[ V = 4447.247569 \]
\[ V \approx 4447.25 \]

4. You start an account with $2500 and an interest rate of 6.5% compounded yearly. How much is in the account after 7 years?

\[ V = 2500 \times (1 + 0.065)^7 \]
\[ V = 3884.964265 \]
\[ V \approx 3884.97 \]

5. A newly hatched channel catfish typically weighs about 0.06 gram. During the first 6 weeks of life, its weight increases by about 10% each day. Write a function to model the situation. How much does the catfish weigh after 6 weeks?

\[ V = 0.06 \times (1 + 0.10)^{42} \]
\[ V = 3.285821954 \]
6. Solve the following equations by using common bases.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>a) (5^{3x+2} = 25) &amp; b) (\left(\frac{1}{16}\right)^{2-3x} = 32^{x+4}) &amp; c) (3^x = 9^{x-1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt[3]{5^2} = 5) &amp; ((10^{-1})^3 = 3a^2x + 4) &amp; (x = \sqrt{x-2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3x + 2 = a) &amp; ((2^{-3})^2 = (2^x)^{x+4}) &amp; (-ax = -ax)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3x = 0) &amp; (-8 + 12x = 5x + 20) &amp; (x = -2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x = 0) &amp; (7x = 28) &amp; (x = 2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Expand each logarithm.

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>a) (\log_a x^2y^3z) &amp; b) (\log_a \frac{xy^2}{z^3})</td>
<td></td>
</tr>
<tr>
<td>(2\log_b x + 3\log_b y + \log_b z) &amp; ((\log_a x + 2\log_a y) - (3\log_a z))</td>
<td></td>
</tr>
</tbody>
</table>

8. Condense each logarithm.

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>a) (\log_a 2x + 3(\log_a x - \log_a y)) &amp; b) (\log_a x^2 - 2\log_a \sqrt{x})</td>
<td></td>
</tr>
<tr>
<td>(2\log_a 2x - (\frac{x^3}{y})) &amp; (\log_a \frac{x^2}{\sqrt{x}})</td>
<td></td>
</tr>
<tr>
<td>(\log_a 2x \cdot \frac{x^3}{y^3}) &amp; (\log_a \frac{x^2}{\sqrt{x}})</td>
<td></td>
</tr>
<tr>
<td>(\log_a \frac{x^2}{y^3}) &amp; (\log_a x)</td>
<td></td>
</tr>
</tbody>
</table>
9. Solve each equation. Check your solution(s).

a) \( \log_{10}(2x+5) = \log_{10}(5x-4) \)
   \[ 2x + 5 = 5x - 4 \]
   \[ 9 = 3x \]
   \[ x = 3 \]
   \[ \checkmark \]

b) \( \log_3(4x+5) - \log_3(3-2x) = 2 \)
   \[ \log_3 \left( \frac{4x+5}{3-2x} \right) = 2 \]
   \[ \frac{9}{1} = \frac{4x+5}{3-2x} \]
   \[ 4x + 5 = 9 - 18x \]
   \[ 22x = -4 \]
   \[ x = -\frac{4}{22} \]
   \[ x = -\frac{2}{11} \]
   \[ \checkmark \]

c) \( \log_2(x+3) + \log_2(x-4) = 3 \)
   \[ \log_\alpha (x+3)(x-4) = 3 \]
   \[ (x+3)(x-4) = 2^3 \]
   \[ x^2 - x - 12 = 0 \]
   \[ x^2 - x - 12 = 0 \]
   \[ (x-4)(x+3) = 0 \]
   \[ x = 4 \]
   \[ \checkmark \]

d) \( \log_9 x + \log_9(x-2) = \log_9 3 \)
   \[ \log_9 x \cdot (x-2) = \log_9 3 \]
   \[ x \cdot (x-2) = 3 \]
   \[ x^2 - 2x - 3 = 0 \]
   \[ (x-3)(x+1) = 0 \]
   \[ x = 3 \]
   \[ x = -1 \]

f) \( 17 = 3^x \)
   \[ 17 = 3^x \]
   \[ \log_3 17 = x \log_3 3 \]
   \[ \frac{\log_3 17}{\log_3 3} = x \]
   \[ x = 2.578901923 \]

e) \( 29 + 10^{10} = 94 \)
   \[ 10^{(r+12)} = 45 \]
   \[ (r+12) \log_{10} 10 = 10 \log_{10} 45 \]
   \[ r + 12 = 1.053212514 \]
   \[ r = -10.134878749 \]

g) \( 8^2 = -6 \)
   \[ t \log_8 8 = \log_8 (-6) \]
   \[ \frac{10^g}{10^g} = \log_8 (-6) \]
   \[ \text{NO SOLUTION} \]

h) \( \log_2(t+1) - \log_2(t-1) = 3 \)
   \[ \log_2 \left( \frac{t+1}{t-1} \right) = 3 \]
   \[ \frac{t+1}{t-1} = 8 \]
   \[ \frac{t+1}{1} = \frac{8}{t-1} \]
   \[ 8 - 8 = t + 1 \]
   \[ t = 9 \]
   \[ \frac{9}{t} = \frac{9}{7} \]