Final Review

1. Find the average rate of change of \( f(x) = 3x^2 + 1 \) between the following points:
   a. \((1, f(1))\) and \((2, f(2))\)
      \[ \frac{13 - 4}{2 - 1} = \frac{9}{1} = 9 \]
   b. \((f(3), f(5))\) and \((m, f(m))\)
      \[ \frac{3m^2 + 1 - (3^2, 1)}{m - 3} = \frac{3m^2 - 3^2}{m - 3} = \frac{3(m - 3)}{m - 3} = 3 \]
   c. \((x, f(x))\) and \((x + h, f(x + h))\)
      \[ \frac{6x^2 + 12x + 3}{6x^2 + 6xh + 3h^2} = \frac{6x^2 + 12x + 3}{6x + 3h} = \frac{6x^2 + 3h^2}{6x + 3h} \]

2. Does the table below represent a function? Do either represent a linear function? If so, write the formula.
   a. Both are functions.
   b. 
      \[ \begin{array}{c|c|c|c|c|c|}
        \hline
        x & 200 & 230 & 300 & 320 & 400 \\
        f(x) & 70 & 68.5 & 65 & 64 & 60 \\
        \hline
      \end{array} \]
      \[ \text{Constant R.O.C.} \Rightarrow m = -0.05 \]

3. State whether the following lines are perpendicular, parallel, or neither.
   a. \( y = 3x + 3 \)
      \[ m_1 = 3 \]
   b. \( y = \frac{1}{3}x + 3 \)
      \[ m_2 = -\frac{3}{1} \]
   c. \( y = 5x + 2 \)
      \[ m_3 = 5 \]
   d. \( y = 2x + 5 \)
      \[ m_4 = 2 \]
   e. \( y = 7x + \frac{1}{3} \)
      \[ m_5 = 7 \]
   f. \( 9y = -7x \)
      \[ m_6 = -\frac{7}{9} \]
   g. \( 9y = 77 - \frac{3}{7}x \)
      \[ m_7 = -\frac{3}{7} \]
4. You have zero dollars now and the average rate of change in your net worth is $5000 per year.
   a. Write a formula that models the above situation. 
      \[ y = 5000x \]
   b. How much money will you have in forty years?
      \[ 5000(40) = 200000 \]
   c. How many years before you have $100,000?
      \[ \frac{5000x}{10000} = \frac{100000}{5000} \]
      \[ x = 20 \]

5. Your gym charges you $35 a month for your membership. They charge you $11.50 per Zumba class you choose to take.
   a. Write a formula that models the above situation. \( \text{for each month?} \)
      \[ y = 35 + 11.50x \]
   b. How much will it cost you this month if you decide to take 5 Zumba classes?
      \[ 35 + 11.50(5) = 82.50 \]
   c. How many Zumba classes did you take if your monthly gym cost was $127?
      \[ 127 = 35 + 11.50x \]
      \[ x = 8 \]

6. Find the equation of the line parallel to \( 3x + 5y = 6 \) and passes through the point \( (0,6) \).
   \[ m = \frac{-3x + 6}{5} \]
   \[ m = \frac{-3}{5} \]
   \[ y = \frac{-3}{5}x + 6 \]

7. Find the equation of the line shown in the diagram below.
   \[ m = \frac{6-2}{5-1} = \frac{4}{4} = 1 \]
   \[ y = x + 1 \]
   \[ b = (0,1) \]
8. Graph the following piecewise function!

\[ f(x) = \begin{cases} 
  x^2 & \text{if } x < 2 \\
  6 & \text{if } x = 2 \\
  10 - x & \text{if } x > 2 \text{ and } x \leq 6
\end{cases} \]

9. Suppose you plan to buy many blank compact disks. You check price lists and find out that if you buy a 1000 CD’s or less you pay $0.74 each. However, if you buy between 1000 and 2000 CD’s the price drops to $0.69 each for the second thousand. Also, for any purchase of more than 2000, the price for the CD’s drops again to $0.64 for each after the 2000th.

a) Create the piecewise function to calculate the cost for purchasing up to 5000 CD’s. Let \( p(n) \) be the cost to purchase \( n \) cd’s.

\[
p(n) = \begin{cases} 
  7.4n & \text{for } 0 \leq n \leq 1000 \\
  6.9(n-1000) + 740 & \text{for } 1000 < n \leq 2000 \\
  6.4(n-2000) + 1430 & \text{for } n > 2000
\end{cases}
\]

b) Calculate the cost to buy 250, 1000, 2000, and 3500 CD's respectively.

\[
p(n) = \begin{cases} 
  7.4n & \text{for } (0, 1000] \\
  6.9n + 50 & \text{for } (1000, 2000] \\
  6.4n + 1430 & \text{for } (2000, \infty)
\end{cases}
\]
10. Write the piecewise function of the graph shown below:

\[ y = \begin{cases} 
X + 2, & X \in [-7, -2] \\
- \frac{4}{3}X - \frac{8}{3}, & X \in (-2, 4] \\
\frac{9}{2}X - 2, & X \in (4, 8] 
\end{cases} \]

11. A phone company charges $29.95 a month for the first 500 minutes and $0.07 for each additional minute up to 1000 minutes. Write a piecewise function, \( C(x) \), of cost as a function of the number of minutes, \( x \), with the appropriate domains.

\[ C(x) = \begin{cases} 
29.95, & (0, 500] \\
0.07(x - 500) + 29.95, & (500, \infty) 
\end{cases} \]

12. 

a. Sketch a graph that models exponential growth

b. Sketch a graph that models exponential decay

13. Without a calculator, match the graph to the appropriate function:

- a) \( 5(4)^x \)
- b) \( 20(0.95)^x \)
- c) \( 15(2.2)^x \)
- d) \( 15(0.5)^x \)
- e) \( 20(4.5)^x \)
14. The cost of a monthly LIRR ticket is about $225. Assume that the price of the ticket rises by 3.45% each year, write a formula that describes the cost of the ticket, \( C \), as a function of time, \( t \).

\[ C(t) = 225 \left(1.0345\right)^t \]

15. The amount of a drug (in milligrams) that remains in the body after \( t \) hours is given by \( A(t) = 50(0.85)^t \).
   
   a. What is the initial dose of the drug given? 50
   
   b. What percent of the drug leaves the body each hour? 15\%
   
   c. What is the amount of the drug after 7 hours? (nearest hundredth of a milligram) 6.03

\[ 50 \left(0.85\right)^7 = 6.03 \]

16. How does an exponential function differ from a linear function?

   Constant % change rather than constant rate of change.

17. Based on the table below, determine which function is linear and which is exponential. Determine a formula for each.

<table>
<thead>
<tr>
<th>( x )</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>30</td>
<td>45</td>
<td>60</td>
<td>75</td>
<td>90</td>
<td>105</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>1000</td>
<td>1200</td>
<td>1440</td>
<td>1728</td>
<td>2073.6</td>
<td>2488.32</td>
</tr>
</tbody>
</table>

   \( f(x) \) is linear, \( g(x) \) is exponential.

18. Write an equation for the graph shown below:

   a. \[ y = 3 \left( \frac{1}{3} \right)^x \]

   b. \[ y = 3 \left( \frac{1}{2} \right)^x \]
Expand terms of $\log x$:

19. $x = \frac{ab^2}{\sqrt{cd}}$ 
   $\log a + 2 \log b - \frac{1}{4} \log c - \frac{1}{3} \log d$

20. $x = \sqrt[3]{\frac{a^2}{bd}}$ 
   $\frac{z}{3} \log a - \frac{1}{3} \log b - \frac{1}{3} \log d$

21. $x = \sqrt[3]{\frac{b^2}{cd}}$ 
   $\frac{1}{2} \log a + 2 \log b - \log c - \log d$

Express as a single log:

22. $\log x = \frac{1}{5} \left( \log m + 2 \log n \right) - \left( 2 \log p + \log q \right)$
   $\sqrt[5]{\frac{m n^2}{p^2 q}}$

23. $\log x = \left( 3 \log a + \log b \right) - \left( \frac{1}{2} \log c + 3 \log d \right)$
   $\frac{a^3 b}{\sqrt{c d^3}}$

24. $\log x = 4 \log a - 2 \log b$ 
   $\frac{a^4}{b^2}$

Express the following in terms of $a$ and $b$ if $\log 3 = a$ and $\log 5 = b$

25. $\log 25$
   $\log a^2 = 2 \log 5 = 2b$

26. $\log 9$
   $\log a^3 = 3 \log 3 = 3a$

27. $\log 45$
   $\log a^4 + \log 5 = 2a + b$
   $\log (a^4 \cdot 5)$
28. Describe the transformation of the function \( f(x) \):

a. \(-f(x)\)

\[
\begin{array}{c}
\downarrow \\
_{x \rightarrow 0}
\end{array}
\]

b. \( f(-x) \)

\[
\begin{array}{c}
\downarrow \\
_{y \rightarrow 0}
\end{array}
\]

c. \( f(x+k) \)

\[
\begin{array}{c}
\rightarrow k
\end{array}
\]

d. \( f(x-k) \)

\[
\begin{array}{c}
\rightarrow -k
\end{array}
\]

e. \( f(x)+k \)

\[
\begin{array}{c}
\uparrow k
\end{array}
\]

f. \( f(x)-k \)

\[
\begin{array}{c}
\downarrow k
\end{array}
\]

g. \( kf(x) \)

vert. stretch by factor of \( k \).

h. \( f(kx) \)

horiz. stretch by factor of \( k \).

29. If \( f(-x) = f(x) \), then the function has ___ ______________ symmetry.

30. If \( f(-x) = -f(x) \), then the function has ___ ______ symmetry. @ (0,0)

State whether the following functions are even, odd, or neither.

31. \( f(x) = \frac{1}{x^2} \)

Even

32. \( f(x) = x + 3 \)

Neither

33. \( f(x) = x^2 + 2x \)

Neither

34. \( f(x) = |x| \)

Even

Let \( f(x) = 1 - x \). Evaluate and simplify:

35. \( f(2x) \)

\[
\frac{1-2x}{x}
\]

36. \( f(x+1) \)

\[
1 - (x+1)
\]

37. \( f(1-x) \)

\[
1 - (1-x)
\]

\[
|\overrightarrow{x}|
\]
Find a formula for a parabola given the following information:

41. Roots at $x = -1, x = 3$ and passes through the point $(0, -3)$.
   
   \[ y = a(x + 1)(x - 3) \]
   
   \[ -3 = \frac{-3}{3} \]
   
   \[ a = 1 \]
   
   \[ y = (x + 1)(x - 3) \]

42. Root at $x = -15$ and the vertex point of $(-6, 9)$
   
   \( f(x, 0) \)
   
   \[ y = a(x + 6)^2 + 9 \]
   
   \[ 9 = a(0)^2 + 9 \]
   
   \[ a = \frac{9}{9} \]
   
   \[ y = -1(x + 6)^2 + 9 \]

43. Roots at $x = 2$ (multiplicity of 2) and a $y$-intercept at $-4$.
   
   \( f(x, 0) \)
   
   \[ y = a(x - 2)^2 \]
   
   \[ -4 = a(0)^2 \]
   
   \[ a = \frac{-4}{-4} \]
   
   \[ y = -(x - 2)^2 \]

44. Fill in the table below: If you do not have enough information, please fill in the cell with a U.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>26</td>
<td>6</td>
<td>-6</td>
<td>-10</td>
<td>-6</td>
<td>6</td>
<td>26</td>
</tr>
<tr>
<td>$f(-x)$</td>
<td>-6</td>
<td>-16</td>
<td>-6</td>
<td>6</td>
<td>6</td>
<td>-6</td>
<td>-26</td>
</tr>
<tr>
<td>$-f(x)$</td>
<td>-26</td>
<td>-6</td>
<td>6</td>
<td>16</td>
<td>6</td>
<td>-6</td>
<td>-26</td>
</tr>
<tr>
<td>$3f(x)$</td>
<td>78</td>
<td>18</td>
<td>-18</td>
<td>-30</td>
<td>-18</td>
<td>18</td>
<td>78</td>
</tr>
<tr>
<td>$f(x+2)$</td>
<td>26</td>
<td>-6</td>
<td>-16</td>
<td>-16</td>
<td>-6</td>
<td>-6</td>
<td>26</td>
</tr>
<tr>
<td>$f(x)+1$</td>
<td>27</td>
<td>7</td>
<td>-9</td>
<td>-5</td>
<td>7</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>$f(2x)$</td>
<td>72</td>
<td>46</td>
<td>-46</td>
<td>-46</td>
<td>-46</td>
<td>46</td>
<td>-46</td>
</tr>
</tbody>
</table>

a) Is $f(x)$ even, odd, or neither?  

$N_e_i_t_h_e_r$ 

$f(-x) \neq \pm f(x)$
45. Fill in the table below. You must know these values.

<table>
<thead>
<tr>
<th></th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin θ</td>
<td>(\frac{1}{2})</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
</tr>
<tr>
<td>cos θ</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>tan θ</td>
<td>(\frac{1}{\sqrt{3}})</td>
<td>1</td>
<td>(\sqrt{3})</td>
</tr>
</tbody>
</table>

46. If tan θ < 0 and csc θ > 0, in which quadrant does θ terminate?

47. Express 160° in radian measure.

\[\frac{160\pi}{180} = \frac{8\pi}{9}\]

48. The exact value of \(\sin \frac{3\pi}{2} - \cos \frac{\pi}{3}\) is

\[-1 - \frac{1}{2} = -1.5\]

49. An angle that measures \(\frac{5\pi}{6}\) radians is drawn in standard position. In which quadrant does the terminal side lie?

50. An art student wants to make a string collage by connecting six equally spaced points on the circumference of a circle to its center with string. What would be the radian measure of the angle between two adjacent pieces of string, in simplest form?

\[\frac{360^\circ}{6} = 6^\circ \Rightarrow \frac{\pi}{3}\]

51. State an angle that is coterminal with 125°?

52. If \(\theta\) is an angle in standard position and \(P(-3,4)\) is a point on the terminal side of \(\theta\), what is the value of sin \(\theta\)?
53. What is the value of \( \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) \)?

\[ \frac{5\pi}{6} \text{ or } 150^\circ \]

54. If \( \sec \theta = -\frac{5}{4} \) and \( \theta \) lies in Quadrant II, what is the value of \( \tan \theta \)?

\[ \cos \theta = -\frac{-4}{5} \]

55. In the accompanying diagram of a unit circle, the ordered pair \((x,y)\) represents the point where the terminal side of \( \theta \) intersects the unit circle. If \( x = -\frac{1}{2} \), what is one possible value for \( \theta \)?

\[ \cos \theta = -\frac{1}{2} \]

56. What is the period of the function \( y = -3 \sin 2x \)?

\[ \frac{2\pi}{2} = \pi \]

57. For which value of \( x \) is the fraction \( \frac{6}{\sin x - 1} \) undefined?

\[ \sin x - 1 = 0 \]

\[ \sin x = 1 \]

\[ x = \frac{\pi}{2} \]
58. Assume that a child starts swinging until she reaches the highest she can swing and keeps her effort constant. The height \( h(t) \) of the seat is given by \( h(t) = 2 \sin \left( \frac{\pi}{2} t \right) + 3 \). Assume that \( t = 0 \) (seconds) is when the child passes through the equilibrium (starting) position.

a) Graph the height of the function \( h(t) \) for \( 0 \leq t \leq 8 \)

b) What is the maximum height above the ground reached by the seat of the swing?

5

c) What is the period of the swinging child?

4

d) For how many seconds is the child above 3 feet?

\( \frac{3}{4} \) (2 out of every 4 seconds)

e) What is the height of the child at 6.5 seconds?

1.585786438
59. The accompanying graph shows a trigonometric function. State an equation of this function.

\[ y = -2 \cos(x) \]

or

\[ y = 2 \sin \left( x - \frac{\pi}{2} \right) \]

60. Given the accompanying graph, find the following:

\[ a) \text{ The midline } = 0 \\
 b) \text{ The Amplitude } = 3 \\
 c) \text{ The Period } = \frac{\pi}{3} \\
 d) \text{ The Frequency } = \frac{2\pi}{3} \\
 e) \text{ Write an equation for this equation: } \\
 y = -3 \sin \left( \frac{2\pi}{3} x \right) \]

61. The tide at a boat dock can be modeled by the equation \( y = -2 \cos \left( \frac{\pi}{6} t \right) + 10 \), where \( t \) is the number of hours past noon and \( y \) is the height of the tide, in feet.

a) Using your calculator, graph this equation for one period beginning at \( t = 0 \).

b) What is the maximum height of the tide in feet?

\[ 12 \]

c) When does the maximum height occur? When does the minimum height occur?

\[ \text{Max at } X = 6, \text{ Min at } X = 0, 12 \]

d) What is the average height of the tide?

\[ \text{Avg } \approx 10 \]

e) For how many hours between \( t = 0 \) and \( t = 12 \) is the tide at least 9 feet?

\[ \text{Between 2 and 10 hrs} \]

i.e., for 8 hours
62. The average annual snowfall in a certain region is modeled by the function $S(t) = 20 + 10 \cos \left( \frac{\pi}{5} t \right)$, where $S$ represents the annual snowfall, in inches, and $t$ represents the number of years since 1970.

a) What is the minimum annual snowfall, in inches, for this region?

\[ \mu_{\min} = 20 - 10 = 10 \]

b) In which years, between 1970 and 2000 did the minimum amount of snow fall?

\[ \overline{1975, 1985, 1995} \]

63. The accompanying data represents the rabbit population ($L_2$) over time, $t$, in months ($L_1$) beginning with $t = 1$ representing Jan 1.

a) Plot the data on the accompanying graph.

b) Find the midline, amplitude, period, and frequency for an equation that represents this scatterplot.

\[ k(0 - 1000) \quad \ell_{\bar{x}} = 12 \]

\[ \text{AMPL} = 5200 \quad \text{PER} = \frac{6}{5} \]

c) Write an equation to represent this data using the parameters found above.

\[ y = -5200 \sin \left( \frac{6}{5} x \right) + 1000 \]

d) During which month does this rabbit population first reach 600?

During month 4

\[ 0 = \text{JAN} \]

\[ 1 = \text{FEB} \]

\[ 2 = \text{MAR} \]

\[ 3 = \text{APR} \]

\[ 4 = \text{MAY} \]
64. A Ferris wheel is 50 meters in diameter and boarded from a platform that is 5 meters above the ground. The six o’clock position on the Ferris wheel is level with the loading platform. The wheel completes one full revolution every 8 minutes. You make 2 complete revolutions on the wheel, starting at \( t = 0 \).

a) Sketch \( h = f(t) \), where \( h \) is your height on the Ferris wheel after \( t \) minutes, according to the scenario described above.

![Graph of \( h = f(t) \)](image)

b) Write an equation in the form \( h = f(t) = A \cos(Bt) + D \)

\[
f(t) = -25 \cos\left(\frac{\pi}{4}t\right) + 30
\]

c) Using the equation found in #7, find an alternative equation in the form \( h = f(t) = A \sin(B(t-C)) + D \)

\[
f(t) = +25 \sin\left(\frac{\pi}{4}(t-2)\right) + 30
\]
65. Solve algebraically for all values of $\theta$ in the interval $[0, 2\pi]$ that satisfy the equation \[
\frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta + \cos \theta} = \frac{1}{3} \]
\[
\sin^2 \theta = 1 + \cos \theta \\
\sin^2 \theta = \frac{1}{\cos \theta + \cos \theta} \\
\sin^2 \theta = \frac{1}{\frac{1}{\cos \theta}} \\
\sin^2 \theta = \cos \theta \\
\cos \theta = \sqrt{1 - \sin^2 \theta} \\
\cos \theta = \sqrt{1 - 1} \\
\cos \theta = 0 \\
\cos \theta = -1 \\
\cos \theta = \frac{\pi}{2}, \frac{3\pi}{2} \\
\theta = \frac{\pi}{2}, \frac{3\pi}{2}
\]

66. Find all values of $x$ in the interval $0^\circ < x < 360^\circ$ that satisfy the equation $3\cos x + \sin 2x = 0$
\[
3\cos x + 2\sin x \cos x = 0 \\
\cos x \left(3 + 2\sin x\right) = 0 \\
\cos x = 0 \\
\frac{\pi}{2}, \frac{3\pi}{2} \\
\sin x = \pm 1
\]

67. Find all values of $x$ in the interval $0^\circ \leq x < 360^\circ$ that satisfy the equation $4\cos^2 x - 5\sin x - 5 = 0$. Express your answer to the nearest tenth of a degree.
\[
4\left(1 - \sin^2 x\right) - 5\sin x - 5 = 0 \\
4\sin^2 x - 5\sin x - 5 = 0 \\
-4\sin^2 x - 5\sin x - 1 = 0 \\
4\sin^2 x + 5\sin x + 1 = 0
\]
\[
\begin{align*}
\text{If } a &= 4 \\
\text{If } b &= 5 \\
\text{If } c &= 1 \\
6 - 4\cos^2 x &= 25 - 4(4)(1) \\
\cos^2 x &= \frac{9}{4} \\
\sin x &= \pm \frac{3}{2} \\
\sin x &= -\frac{3}{2} \\
\sin x &= \frac{3}{2} \\
\cos x &= \pm \frac{\sqrt{1 - \left(\frac{3}{2}\right)^2}}{2} \\
\cos x &= \pm \frac{\sqrt{1 - \frac{9}{4}}}{2} \\
\cos x &= \frac{\sqrt{7}}{2} \\
\sin x &= \frac{3\pi}{2} \\
\end{align*}
\]
68. Find all values of $\theta$ in the interval $0^\circ \leq \theta < 360^\circ$ which satisfy the equation $\cos 2\theta = \cos \theta$.

\[2\cos^2 \theta - 1 = \cos \theta\]
\[2\cos^2 \theta - \cos \theta - 1 = 0\]

\[(2 \cos \theta + 1)(\cos \theta - 1) = 0\]

\[\cos \theta = \frac{-1}{2}\]
\[\cos \theta = 1\]

Ref: $\frac{\pi}{3}$

$0, 2\pi$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

$120^\circ, 240^\circ, 0^\circ, 360^\circ$

69. Find, to the nearest tenth of a degree, all values of $\theta$ in the interval $0^\circ \leq \theta < 360^\circ$ that satisfy the equation $4\cos^2 \theta = 3 + 3\sin \theta$.

\[4(1 - \sin^2 \theta) = 3 + 3\sin \theta\]

\[4 - 4\sin^2 \theta = 3 + 3\sin \theta\]

\[0 = 4\sin^2 \theta + 3\sin \theta - 1\]

\[(4\sin \theta - 1)(\sin \theta + 1) = 0\]

\[\sin \theta = \frac{1}{4}\]
\[\sin \theta = -1\]

Ref: $\frac{\pi}{4}$

$\frac{3\pi}{2}$

70. Find all values of $\theta$ in the interval $0^\circ \leq \theta < 360^\circ$ that satisfy the equation $5\sin \theta + 2\cos 2\theta - 3 = 0$. Express your answer to the nearest tenth of a degree.

\[5\sin \theta + 2(1 - 2\sin^2 \theta) - 3 = 0\]

\[5\sin \theta + 2 - 4\sin^2 \theta - 3 = 0\]

\[-4\sin^2 \theta + 5\sin \theta - 1 = 0\]

\[4\sin^2 \theta - 5\sin \theta + 1 = 0\]

\[(4\sin \theta - 1)(\sin \theta - 1) = 0\]

\[\sin \theta = \frac{1}{4}\]
\[\sin \theta = 1\]

Ref: $\frac{\pi}{4}$

$\frac{3\pi}{2}$