Cumulative Review #1

1. a) A bank pays 6% interest, compounded quarterly. If a person presently has $2000 in the bank, how long would it take until there is $2800 in the account?

\[ \frac{2800}{2000} = (1 + \frac{0.06}{4})^t \]

\[ t = \frac{\ln(1.4)}{\ln(1.015)} \approx 5.6478253 \text{ yrs} \]

b) If the bank compounded the interest daily, how long would it take the person from part (a) to get his $2800?

\[ t = \frac{\ln(1.4)}{\ln(1.0015)} \approx 5.6083315 \text{ yrs} \]

c) A different bank pays 5 3/4% interest, compounded continuously. If a person presently has $2000 in the bank, how long would it take until there is $2800 in the account?

\[ \frac{2800}{2000} = e^{0.0575t} \]

\[ t = \frac{\ln(1.4)}{0.0575} \approx 5.8516911 \text{ yrs} \]

2. Solve each equation below exactly. A calculator solution will receive NO CREDIT.

a. \[ 4(1.34)^t = 2 \]

\[ \ln(4) + t \ln(1.34) = \ln(2) \]

\[ \ln(4) = \ln(2^2) = 2 \ln(2) \]

\[ t = \frac{\ln(2)}{\ln(2) - \ln(1.34)} \]

b. Solve the equation, for exact values of \( \theta \) if possible, \( 0 \leq \theta < 2\pi \).

\[ 2 \cos^2 \theta = \sin \theta + 1 \]

\[ 2(1 - \sin^2 \theta) = \sin \theta + 1 \]

\[ 2 - 2\sin^2 \theta - \sin \theta - 1 = 0 \]

\[ 2\sin^2 \theta + \sin \theta - 1 = 0 \]

\[ \sin \theta = \frac{1}{2} \]

\[ \theta = \frac{\pi}{6}, \frac{5\pi}{6} \]

3. The decay of the radioactive isotope carbon-14 can be modeled by \( Q = Q_0 e^{-0.000121t} \). How long will it take for any sample of carbon-14 to be reduced by 90% of its initial amount. Round your answer to the nearest year.

\[ 0.9 = e^{-0.000121t} \]

\[ \ln(0.9) = -0.000121t \]

\[ t = \frac{\ln(0.9)}{-0.000121} \approx 19188.26 \text{ yrs} \]

4. a) Find a possible formula, \( f(x) \), to represent the graph of this parabola.

\[ y = \frac{1}{4} (x+2)^2 - 2 \]

b) Find the average rate of change for this parabola on the interval \(-2 \leq x \leq 2\).

\[ \frac{f(2) - f(-2)}{2 - (-2)} = \frac{1}{4} (2+2)^2 - 2 \]

\[ = 1 \]

c) Is this function invertible on a domain of all real numbers?

No

c) Evaluate \( f^{-1}(-1) \): [Diagram indicating where it is invertible]
5. Given: \( y = \frac{2x^2 - 18}{x^2 - 2x - 8} \cdot \frac{z}{(x+3)(x-3)} \cdot \frac{(x-4)(x+1)}{(x-4)(x+1)} \)

a) Find each of the following, and explain how you obtained your answer. (It is not acceptable to state that you looked at the calculator’s graph).

i) vertical asymptote(s) \( x = \frac{1}{2} \pm \frac{1}{2} \)

ii) horizontal asymptote(s) \( y = 2 \)

iii) x-intercept \( x = \pm 3 \)

iv) y-intercept \( \frac{2(3)}{(3)(-5)} = \frac{6}{-15} = -\frac{2}{5} \)

b) Sketch a graph of the function, utilizing the information obtained in part (a). Be sure to indicate the asymptotes and intercepts.

6. Refer to the table given below:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2000</td>
<td>1000</td>
<td>500</td>
<td>250</td>
<td>125</td>
</tr>
</tbody>
</table>

a) Determine whether \( f \) is linear, exponential, or neither. Explain. (Do NOT refer to the correlation coefficient in your explanation.)

\[ f(x) = 2000 \cdot \left( \frac{3}{5} \right)^x \]

b) Find a formula for the function \( f(x) \).

\[ a = 2000 \quad y = 2000 \cdot (b)^x \]

\[ \frac{1000}{2000} = \frac{2000}{2000} \cdot \left( \frac{3}{5} \right)^1 \]

\[ \frac{1}{2} = b \]