Name: Answers  Precalculus Chapter 2 Test Review

1) Find the domain and range of each of the following functions:
   a) \( f(x) = \frac{|x|}{x} \)  
      Domain: \( x \neq 0 \)  
      Range: \( \{-1, 1\} \)

   b) \( y = \frac{1}{\sqrt{x+a}} \) where \( a \) is a constant  
      Domain: \( x > -a \)  
      Range: \( y > 0 \) (Restricted Domain!)

   c) If \( g(x) = \frac{ax+c}{ex-d} \) find \( g^{-1}(x) \). What is the domain of \( g^{-1}(x) \)? What is the range of \( g^{-1}(x) \)?

2) \( f(x) = \begin{cases} 
(x-3)^2 & \text{if } x \leq 3 \\
-5 & \text{if } x = 3 \\
-x-3 & \text{if } x > 3 
\end{cases} \)

   a) Find \( f(5) = -8 \)

   b) Is this function invertible? Yes, \( x > 3 \) \( x \neq 3 \)

   c) If \( f \) is invertible then evaluate \( f^{-1}(-4) \)

   d) Sketch the graph of \( f \) for \( 0 \leq x \leq 10 \).  
      (Make clear what happens at \( x = 3 \) and \( x = 6 \)).

   d) Evaluate the following:

      \[ \lim_{x \to 3^-} f(x) = \]  
      \[ \lim_{x \to 3} f(x) = -10 \]  
      \[ \lim_{x \to 3^+} f(x) = \text{DNE} \]
3) Evaluate the following limits:

\[
\lim_{x \to 0} \frac{7x^3 - 13x^2 + 4}{3x^3 - 2x^2} = \frac{-13x^2}{3x^2} = \frac{-13}{3}
\]

\[
\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{(x-2)(x-1)}{(x-2)(x+3)} \Rightarrow \frac{(2-1)}{(2+3)} = \frac{1}{5}
\]

\[
\lim_{x \to 2} \frac{2-2^{1/2}}{x} = \lim_{x \to \infty} e^{2-\frac{x}{2}} = e^{2-1} = e^1 = e
\]

4) A phone data plan charges $49 for any data usage up to 10Gb, and then $10 for each additional Gb.
   a) Write a formula for the cost, \( C(d) \), as a function of data usage, \( d \), in Gb.

\[
C(d) = \begin{cases} 
49, & 0 < d \leq 10 \\
10(d-10) + 49, & d > 10 
\end{cases}
\]

b) Sketch the function.

\[\text{Cost} \quad \begin{array}{c}
\$49 \\
(10, 49) \\
\end{array} \]

\[\text{data \ (Gb)} \]

\[\begin{array}{c}
10 \\
\end{array} \]

d) How much does it cost to use 32 Gb in a month?

\[
C(32) = 10(22-10) + 49 = 220 + 49 = 269
\]

e) How many Gbs are you using if you are charged $75 (to the nearest tenth of a Gb?)

\[
\frac{75}{10} = \frac{10(x-10) + 49}{10} \Rightarrow \frac{75}{10} = X - 10 + 49
\]

\[
\frac{x-10}{10} = \frac{2.6}{10} + 10
\]

\[
x = 12.6 \text{ Gb}
\]