### Breaking Down Rational Functions

**Without a Calculator**

#### Given Function
\[ y = \frac{k(x^2 + ax + bx + cb)}{k(x-a)(x-b)(x-c)(x-d)} \]

#### x-Intercepts:
\[ (x-a)(x-b) = 0 \]
\[ x = a, \quad x = b \]

\[ (a,0), \quad (b,0) \]

#### y-Intercept:
\[ y = \frac{k(0-a)(0-b)}{k(0-c)(0-d)} \]
\[ y = \frac{kab}{c} \quad \text{(constants)} \]

#### Vertical Asymptote(s):
\[ (x-c)(x-d) = 0 \]
\[ x = c, \quad x = d \]

#### Horizontal Asymptote:
\[ \text{(End-run behavior)} \]
\[ y = \frac{k(x)(x)}{(x)(x)} \Rightarrow \frac{kx^2}{x^2} \]
\[ y = k \]

#### Holes:
\[ \text{NOT YET! (on THUR)} \]

**With a Calculator**

#### Go to TOV

Where \( y = 0 \).

(Only good to find integers)

#### Go to TOV

at \( x = 0 \)

find \( y \) int.

#### Look for

"Error" on TOV (only integers)

#### Change TABLE START

to 1,000,000

Check TOV for \( y \) value
Horiz Asymptotes: \( f(x) = \frac{p(x)}{g(x)} \)

I. Degree of \( p(x) \) < Degree of \( g(x) \)

- ALWAYS HAS \( y = 0 \) as the horizontal asymptote.

- Example:
  \[ y = \frac{3x^2 - 2x + 5}{4x^3 + 5x^2 + ...} \]
  \[ y = \frac{3x^2}{4x^3} \]
  \[ \lim_{x \to \infty} y = 0 \]

II. Degree of \( p(x) = g(x) \)

- ALWAYS REDUCE TO COEFFICIENTS OF LEADING TERMS.

- Example:
  \[ y = \frac{7x^4}{3x^4} \]
  \[ y = \frac{7}{3} \]

III. Degree of \( p(x) > g(x) \)

- NO Horiz Asy!

- Example:
  \[ y = \frac{5x^3 + 2x^2 + ...}{4x^3} \]
  \[ y = \frac{5x^3}{4x^3} \]
  \[ \lim_{x \to \infty} y = \infty \]
  \[ y \to +\infty \]
Putting it all together:

Identify the key features of the given function. Also include a sketch of the function that includes the intercepts and asymptotes drawn on the graph.

\[
y = \frac{x^2 - 4x + 6}{x^3 + x - 6}
\]

[Show factored form here:]

\[
f(x) = \frac{x(x - 4)}{(x + 3)(x - 2)}
\]

- **x-intercepts:** (nominator = 0)
  \(x(x - 4) = 0\)
  \(x = 0, x = 4\)
  \((0,0), (4,0)\)
- **y-intercepts:** (constant term)
  \(y = \frac{6}{6} = 1\) \((0,0)\)
- **Vertical Asymptote:** (Denominator = 0)
  \((x + 3)(x - 2) = 0\)
  \(x = -3, x = 2\)
- **Horizontal Asymptote:** Leading term of numerator
  \(y = \frac{x^2}{x^3} \Rightarrow y = \frac{1}{x}\)

- **x-intercepts:** \((0, 0)\) and \((4, 0)\)
- **y-intercept:** \((0, 0)\)
- **Vertical Asymptote(s):** \(x = -3\) and \(x = 2\)
- **Horizontal Asymptote:** \(y = 1\)
- **Find the coordinates of the “hole”, if any:** NONE YET \(\Rightarrow\) This will change later in the week.
- **Describe the end behavior:**
  As \(x \to \infty\), then \(f(x) \to 1\) "RIGHT"
  Graph approaches \(y = 1\)
  As \(x \to -\infty\), then \(f(x) \to 1\) "LEFT"
  Graph approaches \(y = 1\)

This will be the same value as the HORIZ ASY!