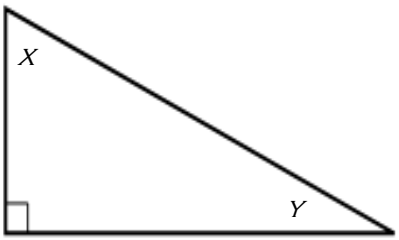
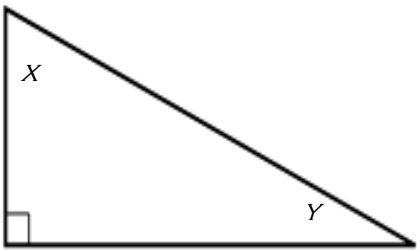
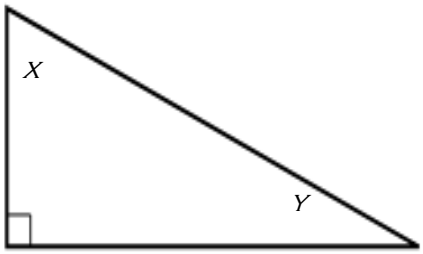
Intro to Trigonometry Ratios

Part I – Identifying the hypotenuse, adjacent side, and opposite side in a right triangle







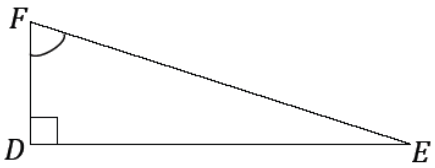
CALCULATORS MUST BE IN DEGREE MODE. GO TO THE MODE BUTTON AND HIGHLIGHT DEGREE.

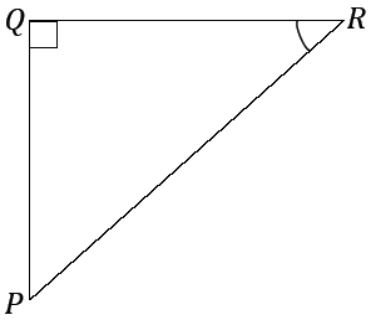
'TRIG ROW' ON THE CALCULATOR IS THE 4TH ROW DOWN

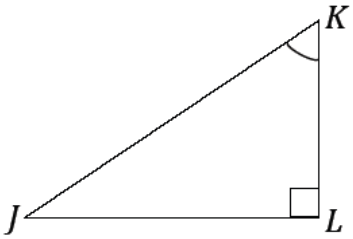
Trigonometry	Sine	Cosine	Tangent
Acronym			
Ratio			
Trig Ratio			

We use these ratios to find missing sides and missing angles in RIGHT triangles!

1. For each exercise, label the appropriate sides as adjacent, opposite, and hypotenuse, with respect to the marked acute angle. Then find he ratios below. Then find the ratios below.

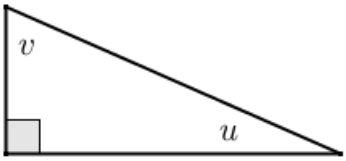






Sine (F)	**Sine (R)**	**Sine (K)**
Cosine (F)	**Cosine (R)**	**Cosine (K)**
Tangent (F)	**Tangent (R)**	**Tangent (K)**

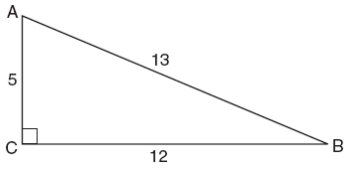
2. If u and v are the measures of complementary angles such that $\sin u = \frac{2}{5}$ and $\tan v = \frac{\sqrt{21}}{2}$, label the sides and angles of the right triangle in the diagram below with possible side lengths.



3. In ΔWXY, the measure of $\measuredangle Y = 90$, XW = 53, YX = 28, and WY = 45. What is the value of the cosine of $\measuredangle X$ to the nearest hundredth? These will be like the DELTA MATH questions.

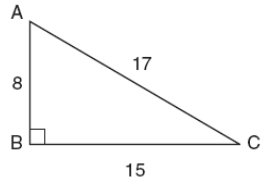
Homework

1. Which ratio represents cos *A* in the accompanying diagram of ΔABC?



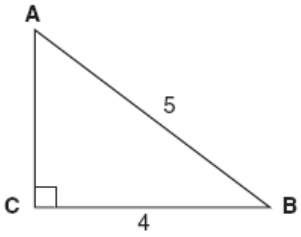
(1) $\frac{5}{13}$ (2) $\frac{12}{5}$ (3) $\frac{12}{13}$ (4) $\frac{13}{5}$

2. In the accompanying diagram of right triangle *ABC*, *AB* = 8, *BC* = 15, *AC* = 17, and m$\angle$ *ABC* = 90. What is tan $\angle$ *C*?



(1) $\frac{8}{15}$ (2) $\frac{8}{17}$ (3) $\frac{17}{15}$ (4) $\frac{15}{17}$

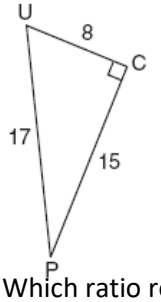
3. Which equation could be used to find the measure of one acute angle in the right triangle shown below?



(1) $\sin A = \frac{4}{5}$ (2) $\cos B = \frac{5}{4}$

(3) $\tan A = \frac{5}{4}$ (4) $\tan B = \frac{4}{5}$

4. The diagram below shows right triangle *UPC*. Which ratio represents the sine with respect to acute $\angle U$?

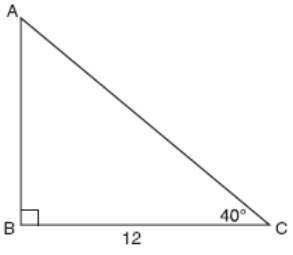


(1) $\frac{15}{8}$ (2) $\frac{8}{15}$ (3) $\frac{15}{17}$ (4) $\frac{8}{17}$

5. In triangle *MCT*, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and *TM* = 13 cm. Which ratio represents the sine with respect to acute $\angle C$?

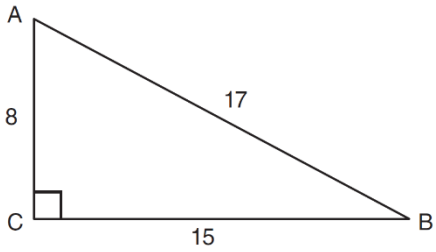
(1) $\frac{13}{85}$ (2) $\frac{13}{84}$ (3) $\frac{84}{85}$ (4) $\frac{84}{13}$

6. In the accompanying diagram of right triangle *ABC*, $BC = 12$ and $m\angle C = 40$. Which single function could be used to find *AB*?



(1) $\tan 50^\circ$ (2) $\cos 40^\circ$ (3) $\sin 50^\circ$ (4) $\sin 40^\circ$

7 Right triangle *ABC* has legs of 8 and 15 and a hypotenuse of 17, as shown in the diagram below. The value of the tangent of $\angle B$ is



1) 0.4706

2) 0.5333

3) 0.8824

4) 1.8750

8 In $\triangle ABC$, the measure of $\angle B = 90°$, $AC = 50$, $AB = 48$, and $BC = 14$. Which ratio represents the tangent of $\angle A$?

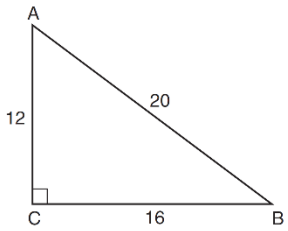
1) $\frac{14}{50}$

2) $\frac{14}{48}$

3) $\frac{48}{50}$

4) $\frac{48}{14}$

9 In right triangle *ABC* shown below, $AC = 12$, $BC = 16$, and $AB = 20$. Which equation is *not* correct?



1) $\cos A = \frac{12}{20}$

2) $\tan A = \frac{16}{12}$

3) $\sin B = \frac{12}{20}$

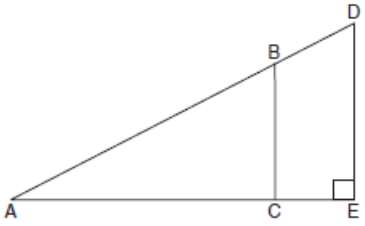
4) $\tan B = \frac{16}{20}$

10 In the diagram of right triangle *ADE* below, $\overline{BC} \parallel \overline{DE}$. Which ratio is always equivalent to the sine of $\angle A$?

1) $\frac{AD}{DE}$

2) $\frac{AE}{AD}$

3) $\frac{BC}{AB}$



4) $\frac{AB}{AC}$

Cofunctions - Sine and Cosine of Complementary Angles

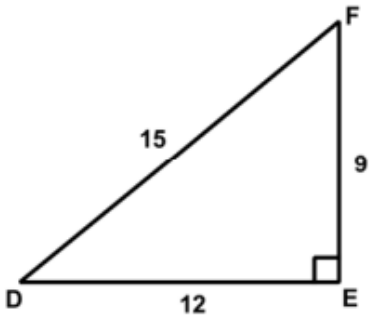
DO NOW: a) Given right triangle DEF shown, write down each of the following ratios. Leave answers as fractions!

(a) $\sin D =$ _____ (b) $\cos D =$ _____

(c) $\tan D =$ _____

(d) $\sin F =$ _____ (e) $\cos F =$ _____

(f) $\tan F =$ _____



b) In any right triangle, what must be true about the sum of the measures of the two acute angles?

c) Do you notice anything about the sine and cosine of the acute angles in (a)? What do you notice?

Find missing angle measures to satisfy cofunctions.

1. What should we do to find a missing angle, if we know one of the two angle measures of a cofunction?

2. Find values for θ that make each statement true.

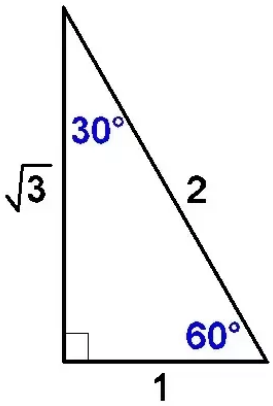
$\sin \theta = \cos (25)$ $\sin 80 = \cos \theta$

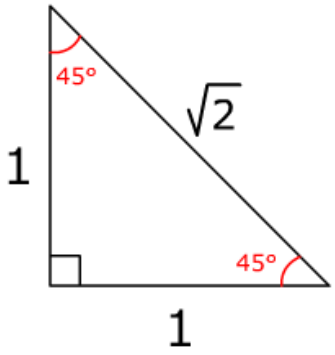
$\sin \theta = \cos (\theta + 10)$ $\sin (\theta - 45) = \cos (\theta)$

3. For what angle measurement must sine and cosine have the same value? Explain how you know.

4. Using your special right triangles from unit 12, fill in the following table.

θ	0°	30°	45°	60°	90°
Sine	0				1
Cosine	1				0





5. A square has side lengths of $5\sqrt{2}$. Use sine or cosine from the table to find the length of the diagonal of the square. Confirm your answer using the Pythagorean Theorem.

6. Given an equilateral triangle with sides of length 12, use sine or cosine from the table to find the length of the altitude. Confirm your answer using the Pythagorean Theorem.

Homework

1. Find the value of θ that makes each statement true.

$\sin\theta = \cos(\theta + 38)$ $\cos\theta = \sin(\theta - 30)$ $\sin\theta = \cos(3\theta + 20)$ $\sin\left(\frac{\theta}{3} + 10\right) = \cos\theta$

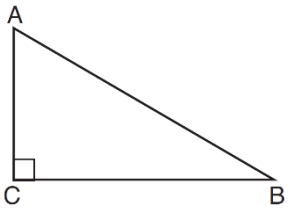
$\sin(A - 30)^\circ = \cos 60^\circ$ $\cos(2x - 1)^\circ = \sin(3x + 6)^\circ$ $\sin(x - 3)^\circ = \cos(2x + 6)^\circ$

2. Landon thinks that the sum $\sin 30 + \sin 30$ is equal to $\sin 60$. Do you agree with Landon? Explain what this means about the sum of the sines of angles.

3. A square has side lengths of $7\sqrt{2}$. Use sine or cosine to find the length of the diagonal of the square. Confirm your answer using the Pythagorean theorem.

4. Given an equilateral triangle with sides of length 9, find the length of the altitude. Confirm your answer using the Pythagorean theorem.

5 In scalene triangle *ABC* shown in the diagram below, $m\angle C = 90°$.



Which equation is always true?

1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$

6 In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?

1) $\tan\angle A = \tan\angle B$
2) $\sin\angle A = \sin\angle B$
3) $\cos\angle A = \tan\angle B$
4) $\sin\angle A = \cos\angle B$

7 Which expression is always equivalent to $\sin x$ when $0° < x < 90°$?

1) $\cos(90° - x)$
2) $\cos(45° - x)$
3) $\cos(2x)$
4) $\cos x$

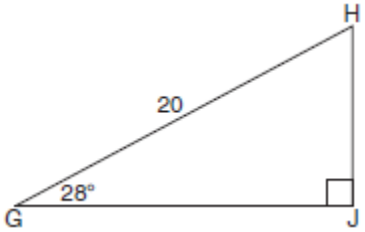
8 In $\triangle ABC$, where $\angle C$ is a right angle, $\cos A = \frac{\sqrt{21}}{5}$. What is $\sin B$?

1) $\frac{\sqrt{21}}{5}$
2) $\frac{\sqrt{21}}{2}$
3) $\frac{2}{5}$
4) $\frac{5}{\sqrt{21}}$

9 In right triangle *ABC* with the right angle at *C*, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of *x*. Explain your answer.

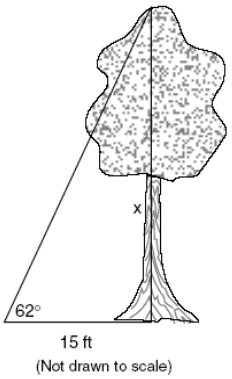
10 Explain why $\cos(x) = \sin(90 - x)$ for *x* such that $0 < x < 90$.

11 When instructed to find the length of $\overline{HJ}$ in right triangle *HJG*, Alex wrote the equation $\sin 28° = \frac{HJ}{20}$ while Marlene wrote $\cos 62° = \frac{HJ}{20}$. Are both students' equations correct? Explain why.



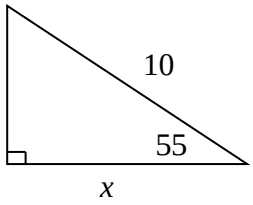
Finding Missing Side Lengths

DO NOW: Find, to the *nearest tenth of a foot*, the height of the tree represented in the accompanying diagram.



Using the appropriate trig ratio, find *x* to the nearest integer.

1.



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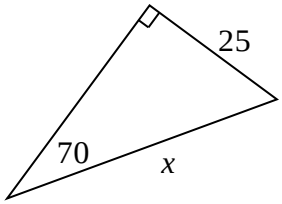

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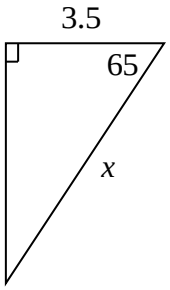

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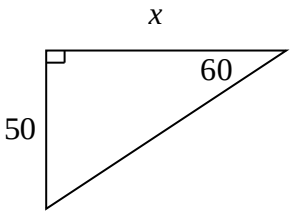

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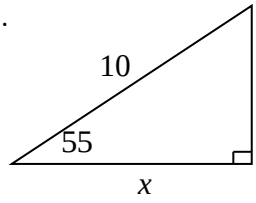

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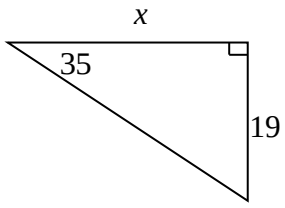
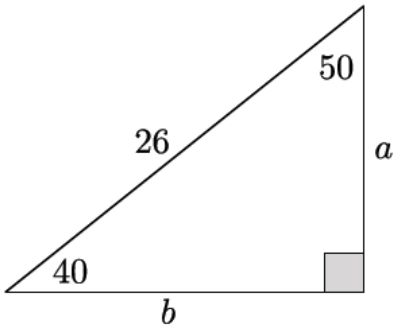
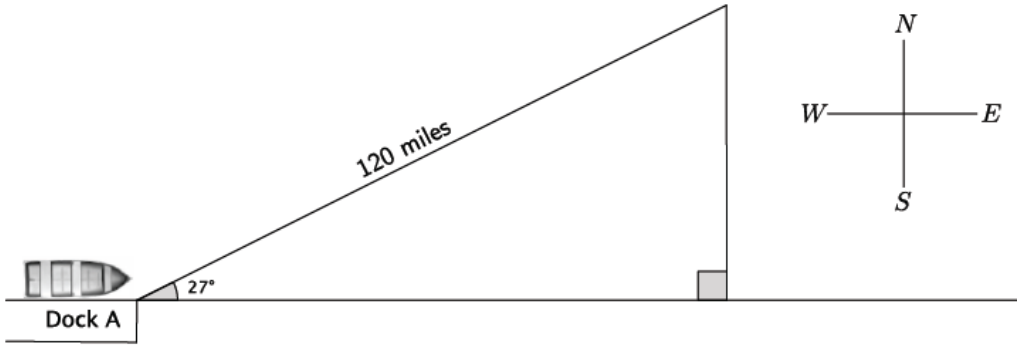


7. Find the values of *a* and *b*. *Round answers to the nearest hundredth.*



8. A shipmate set a boat to sail exactly 27° NE from the dock. After traveling 120 miles, the shipmate realized he had misunderstood the instructions from the captain; he was supposed to set sail going directly east!

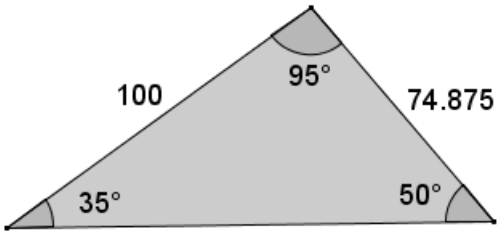


a) How many miles will the shipmate have to travel directly south before he is directly east of the dock? Round your answer to the nearest mile.

b) How many extra miles does the shipmate travel by going the wrong direction compared to going directly east? Round your answer to the nearest mile.

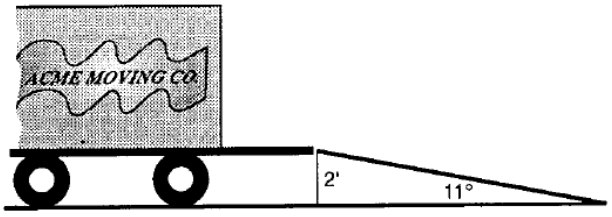
9. Johanna borrowed some tools from a friend so that she could precisely, but not exactly, measure the corner space in her backyard to plant some vegetables. She wants to build a fence to prevent her dog from digging up the seeds that she plants. Johanna returned the tools to her friend before making the most important measurement: the one that would give the length of the fence!

Johanna decided that she could just use the Pythagorean theorem to find the length of the fence she'd need. Is the Pythagorean theorem applicable in this situation? Explain.



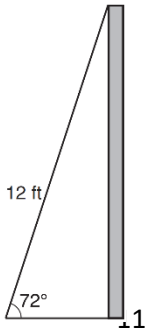
10. The tailgate of a truck is 2 feet above the ground. The incline of a ramp used for loading the truck is 11°, as shown below.

Find, to the *nearest tenth of a foot*, the length of the ramp.



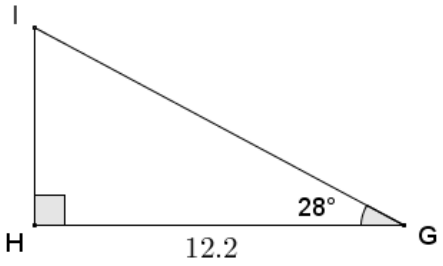
11. As shown in the diagram below, a ladder 12 feet long leans against a wall and makes an angle of 72° with the ground.

Find, to the *nearest tenth of a foot,* the distance from the wall to the base of the ladder.



Homework

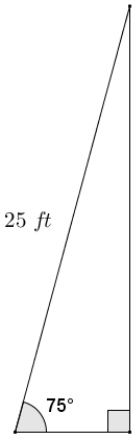
1. Given right triangle GHI, with right angle at H, $GH = 12.2$, and $m\angle G = 28°$, find the measures of the remaining sides and angle to the nearest tenth.



2. The Occupational Safety and Health Administration (OSHA) provides standards for safety at the workplace. A ladder is leaned against a vertical wall according to OSHA standards and forms an angle of approximately $75°$ with the floor. *Round answers to the nearest tenth.*

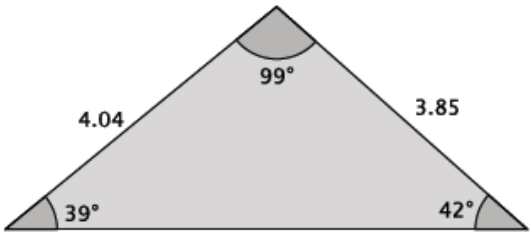
a) If the ladder is 25 ft. long, what is the distance from the base of the ladder to the base of the wall?

b) How high on the wall does the ladder make contact?



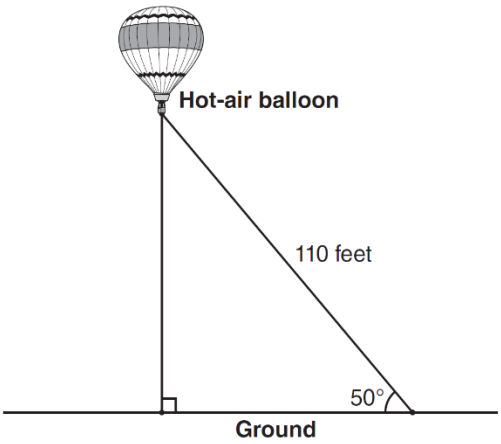
3. The hypotenuse of right triangle *ABC* is 25 and $m\angle A = 41^\circ$. What is the measure, to the nearest tenth, of the leg opposite angle *A*?

4. The measurements of the triangle shown below are rounded to the nearest hundredth. Calculate the missing side length to the nearest hundredth.

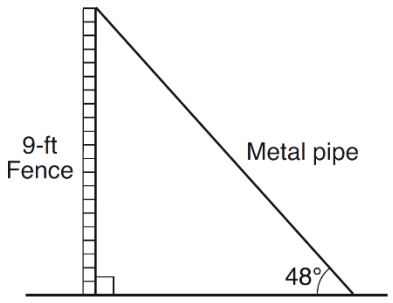


5. A hot-air balloon is tied to the ground with two taut (straight) ropes, as shown in the diagram below. One rope is directly under the balloon and makes a right angle with the ground. The other rope forms an angle of 50º with the ground.

Determine the height, to the *nearest foot,* of the balloon directly above the ground. Determine the distance, to the *nearest foot,* on the ground between the two ropes.

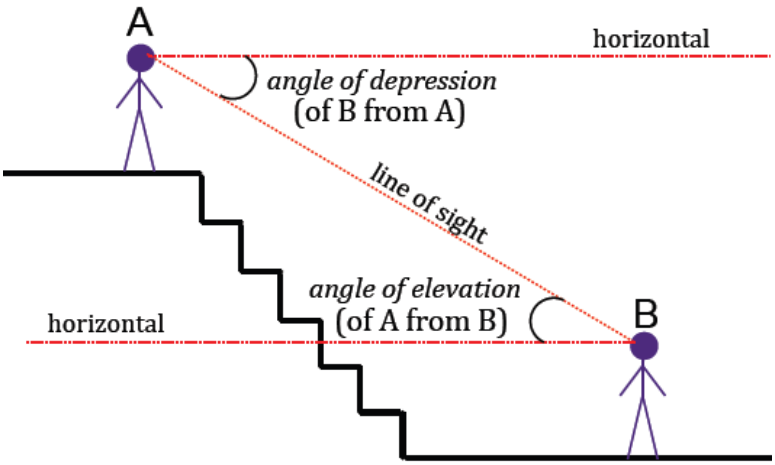


6. A metal pipe is used to hold up a 9-foot fence, as shown in the diagram below. The pipe makes an angle of 48° with the ground. Determine, to the *nearest foot*, how far the bottom of the pipe is from the base of the fence. Determine, to the *nearest foot*, the length of the metal pipe.



Applying Tangents – Angle of Elevation and Depression

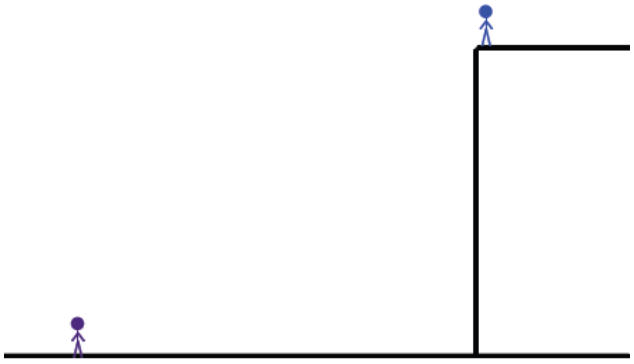
DO NOW: Consider the image below. How would you describe the **angle of elevation**? How would you describe the **angle of depression**?



Examples:

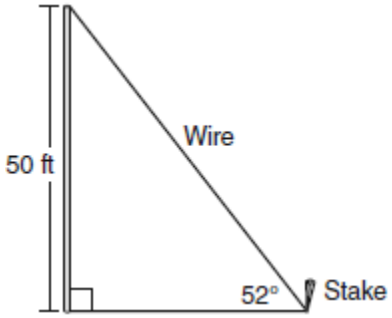
1. Scott, whose eye level is 1.5 m above the ground, stands 30 m from a tree. The angle of elevation of a bird at the top of the tree is 36°. How far above ground is the bird?

2. From an angle of depression of 40°, John watches his friend approach his building while standing on the rooftop. The rooftop is 16 m from the ground, and John's eye level is at about 1.8 m from the rooftop. What is the distance between John's friend and the building?

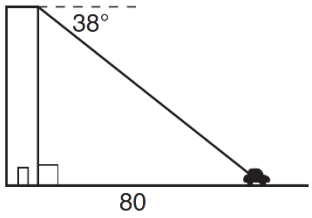


3. A stake is to be driven into the ground away from the base of a 50-foot pole, as shown in the diagram below. A wire from the stake on the ground to the top of the pole is to be installed at an angle of elevation of 52°.

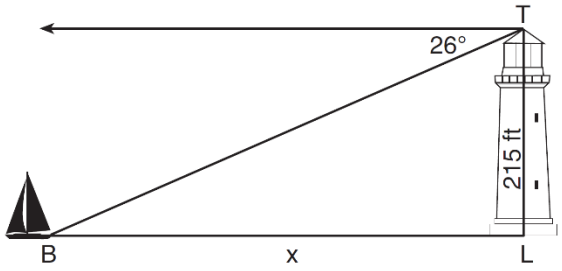
How far away from the base of the pole should the stake be driven in, to the *nearest foot*? What will be the length of the wire from the stake to the top of the pole, to the *nearest foot*?



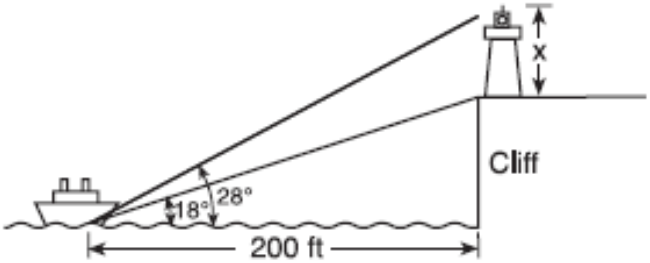
4. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot*.



5. The top of a lighthouse, *T*, is 215 feet above sea level, *L*, as shown in the diagram below. The angle of depression from the top of the lighthouse to a boat, *B*, at sea is 26°. Determine, to the *nearest foot*, the horizontal distance, *x*, from the boat to the base of the lighthouse.

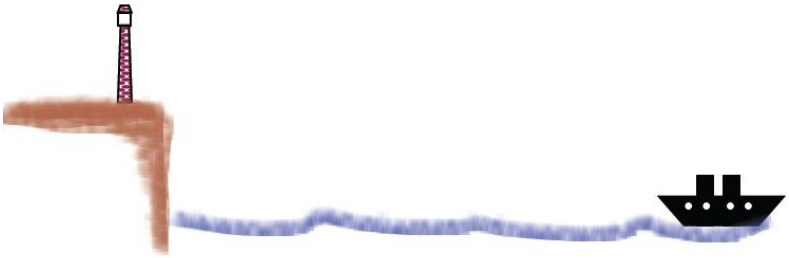


6. A lighthouse is built on the edge of a cliff near the ocean, as shown in the accompanying diagram. From a boat located 200 feet from the base of the cliff, the angle of elevation to the top of the cliff is 18° and the angle of elevation to the top of the lighthouse is 28°. What is the height of the lighthouse, *x*, to the *nearest tenth of a foot*?



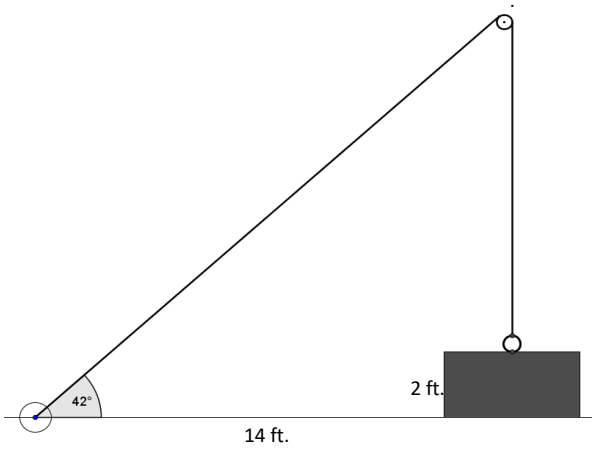
Homework

1. Standing on the gallery of a lighthouse (the deck at the top of a lighthouse), a person spots a ship at an angle of depression of 20°. The lighthouse is 28 m tall and sits on a cliff 45 m tall as measured from sea level. What is the horizontal distance between the lighthouse and the ship? Sketch a diagram to support your answer. *Round answers to the nearest hundredth.*

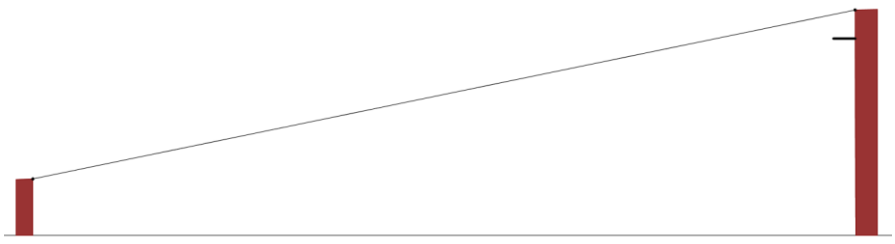


2. An anchor cable supports a vertical utility pole forming a 51° angle with the ground. The cable is attached to the top of the pole. If the distance from the base of the pole to the base of the cable is 5 meters, how tall is the pole? *Round answers to the nearest tenth.*

3. A winch is a tool that rotates a cylinder, around which a cable is wound. When the winch rotates in one direction, it draws the cable in. Joey is using a winch and a pulley (as shown in the diagram) to raise a heavy box off the floor and onto a cart. The box is 2 ft. tall, and the winch is 14 ft. horizontally from where cable drops down vertically from the pulley. The angle of elevation to the pulley is 42°. What is the approximate length of cable required to connect the winch and the box?

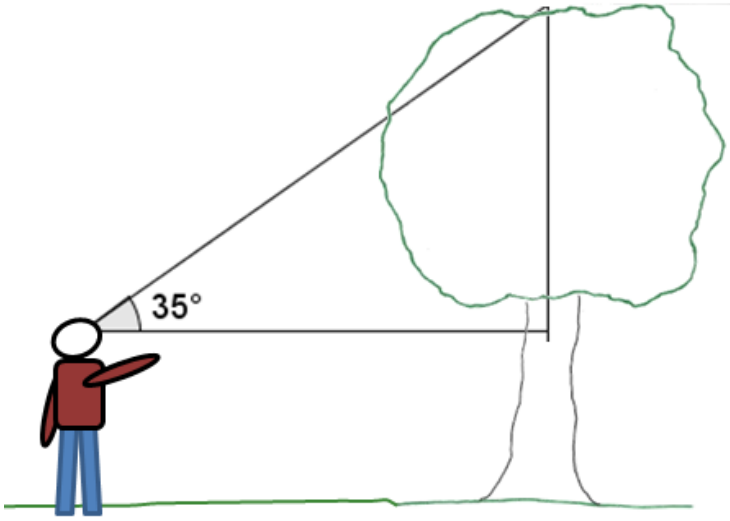


4. Samuel is at the top of a tower and will ride a trolley down a zip-line to a lower tower. The total vertical drop of the zip-line is 40 ft. The zip line's angle of elevation from the lower tower is 11.5°. What is the horizontal distance between the towers?

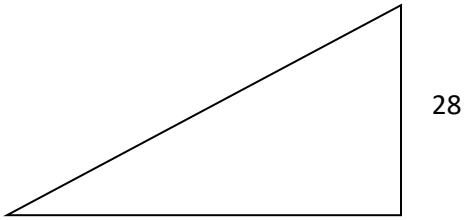


Inverse Trig Functions - Finding Missing Angle Measures

DO NOW: Dan was walking through a forest when he came upon a sizable tree. Dan estimated he was about 40 meters away from a tree when he measured the angle of elevation between the horizontal and the top of the tree to be 35 degrees. If Dan is about 2 meters tall, to the nearest meter, how tall is the tree?

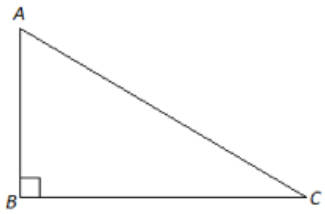


What if we were given a similar example as above except this time, we were asked to find the angle measure? To the nearest degree.



In the same way that mathematicians have named certain ratios within right triangles, they have also developed terminology for identifying angles in a right triangle, given the ratio of the sides. Mathematicians will often use the prefix "arc" to define these because an angle is not just measured as an angle, but also as a length of an *arc* on the unit circle.

Given a right triangle $\triangle ABC$, the measure of angle C can be found in the following ways:

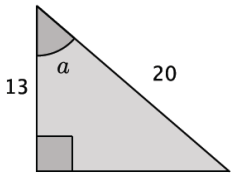


We can write similar statements to determine the measure of angle A.

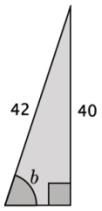
We can use a calculator to help us determine the values of arcsin, arccos, and arctan. Most calculators show these buttons as "$\sin^{-1}$," "$\cos^{-1}$," and "$\tan^{-1}$." This subject will be addressed again in future courses.

1. Find the measure of angles a–d to the nearest degree.

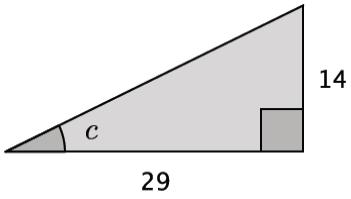
a.



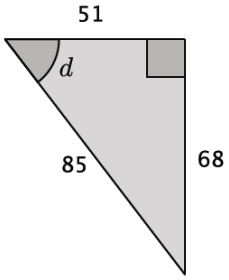
b.



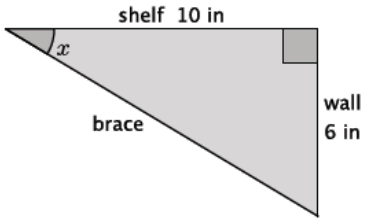
c.



d.



2. Shelves are being built in a classroom to hold textbooks and other supplies. The shelves will extend 10 in from the wall. Support braces will need to be installed to secure the shelves. The braces will be attached to the end of the shelf and secured 6 in below the shelf on the wall. What angle measure will the brace and the shelf make?



3. A 16 ft ladder leans against a wall. The foot of the ladder is 7 ft from the wall. Round answers to the nearest whole number.

a) Find the vertical distance from the ground to the point where the top of the ladder touches the wall.

b) Determine the measure of the angle formed by the ladder and the ground.

4. A group of friends have hiked to the top of the Mile-High Mountain. When they look down, they can see their campsite, which they know is approximately 3 miles from the base of the mountain.

a) Sketch a drawing of the situation.

b) What is the angle of depression?

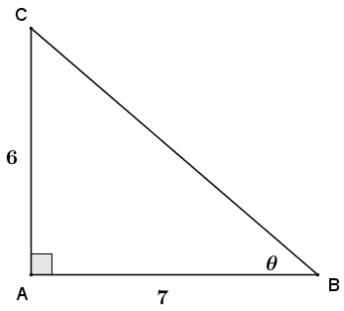
5. A 17-foot ladder is leaning against a wall. The base of the ladder is 5 feet away from the wall. Find to the nearest degree the angle the ladder makes with the wall.

6. A 22-foot pole that is leaning against a wall reaches a point that is 16 feet above the ground. Find to the nearest degree the number of degrees contained in the angle that the pole makes with the ground.

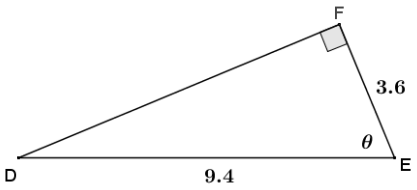
Homework

1. For each triangle shown, use the given information to find the indicated angle to the nearest degree.

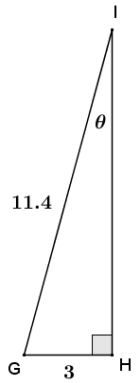
a.



b.



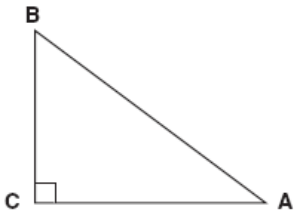
c.



2. S*olving a right triangle* means using given information to find all the angles and side lengths of the triangle. Use $arcsin$ and $arccos$, along with the given information, to solve right triangle ABC if leg $AC = 12$ and hypotenuse $AB = 15$.

Once you have found the measure of one of the acute angles in the right triangle, can you find the measure of the other acute angle using a different method than those used in this lesson? Explain.

3. In the diagram of ΔABC shown below, $BC = 10$ and $AB = 16$. To the *nearest tenth of a degree*, what is the measure of the largest acute angle in the triangle?



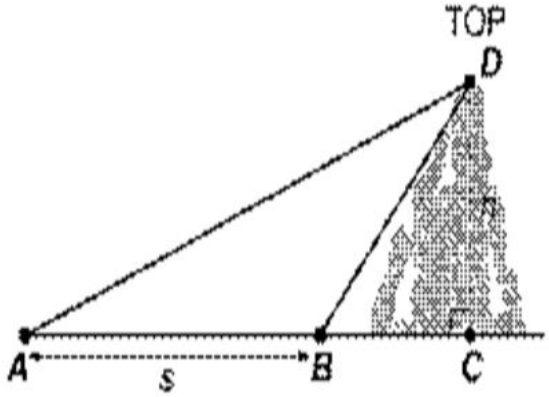
(1) 32.0 (2) 51.3 (3) 38.7 (4) 90.0

Law of Sines

We can use the Law of Sines when we DO NOT have a Right Triangle.

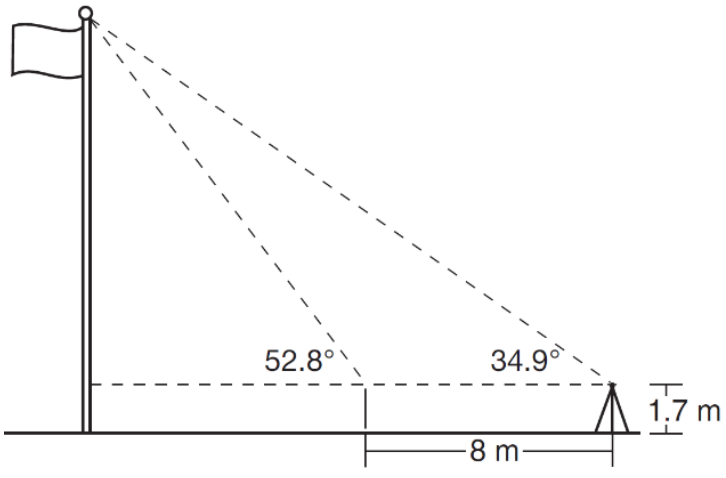
$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC} = \frac{side}{opposite\ angle}$$

1. A ship at sea heads directly towards a cliff on the shoreline. The accompanying diagram shows the top of the cliff, D, sighted from two locations, A and B, separated by a distance of s. If the measure of angle DAC is 30 degrees and the measure of angle DBC is 45 degrees and s = 30feet. What is the height of the cliff, to the nearest foot?

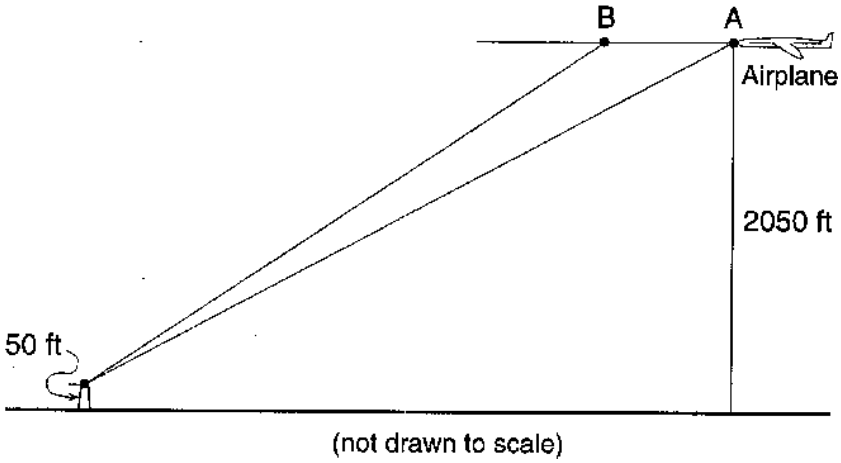


2. Cathy wants to determine the height of the flagpole shown in the diagram below. She uses a survey instrument to measure the angle of elevation to the top of the flagpole and determines it to be 34.9°. She walks 8 meters closer and determines the new measure of the angle of elevation to be 52.8°. At each measurement, the survey instrument is 1.7 meters above the ground.

Determine and state, to the *nearest tenth of a meter*, the height of the flagpole.



3. An airplane traveling at a level altitude of 2050 feet sights the top of a 50-foot tower at an angle of depression of 28º from point *A*. After continuing in level flight to point *B*, the angle of depression to the same tower is 34º. Find, to the *nearest foot*, the distance that the plane traveled from point *A* to point *B*.

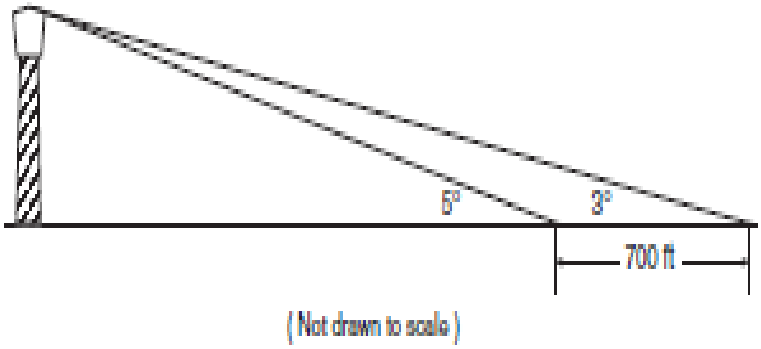


3. A ship captain at sea uses a sextant to sight an angle of elevation of 37° to the top of a lighthouse. After the ship travels 250 feet directly toward the lighthouse, another sighting is made, and the new angle of elevation is 50°. The ship's charts show that there are dangerous rocks 100 feet from the base of the lighthouse. Find, to the *nearest foot,* how close to the rocks the ship is at the time of the second sighting.

Applications

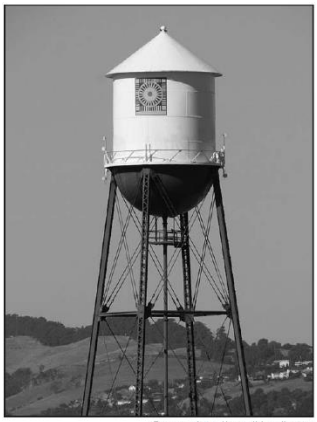
Please attempt questions 2 and 3 for a decent amount of time before you look them up on the world wide web. They were actual questions from previous Regents exams, so they are excellent practice!!

1. While sailing a boat offshore, Donna sees a lighthouse and calculates that the angle of elevation to the top of the lighthouse is 3°, as shown in the accompanying diagram. When she sails her boat 700 feet closer to the lighthouse, she finds that the angle of elevation is now 5°. How tall, to the *nearest tenth of a foot,* is the lighthouse?

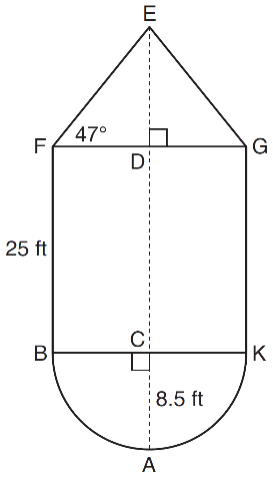


2. Freda, who is training to use a radar system, detects an airplane flying at a constant speed and heading in a straight line to pass directly over her location. She sees the airplane at an angle of elevation of 15° and notes that it is maintaining a constant altitude of 6250 feet. One minute later, she sees the airplane at an angle of elevation of 52°. How far has the airplane traveled, to the *nearest foot*? Determine and state the speed of the airplane, to the *nearest mile per hour*. [This was a 6 pt question on the 2017 regents – YIKES!!!!]

3. The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let *C* be the center of the hemisphere and let *D* be the center of the base of the cone. [This was a 6 pt question on the 2015 exam – OH BOY!!!]



Source: http://en.wikipedia.org



If $AC = 8.5$ feet, $BF = 25$ feet, and $\mathbf{m}\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its max volume and not exceed the limit?