10/30/17 Aim: Definition and Properties of Volume
Do now: Copy the student outcome. Then complete the opening exercise.

Student Outcomes

- Students understand the precise language that describes the properties of volume.
- Students understand that the *volume of any right cylinder* is given by the formula .

Homework TBA

Mar 2-8:24 AM

- A general cone whose vertex lies on the perpendicular line to the base and that passes through the center of the base is a *right cone* (or a *right pyramid* if the base is polygonal). Figure 4 shows a right rectangular pyramid, while Figure 3 shows a rectangular pyramid that is not right.

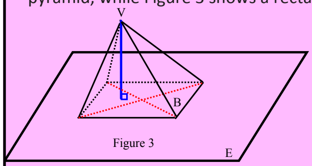


Figure 3

Has a perpendicular but not in the center.

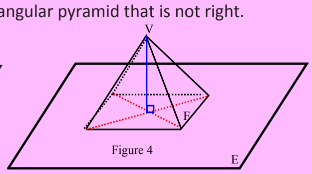
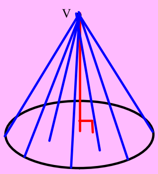


Figure 4

Perpendicular and forms a right angle with the center of the base

Feb 27-6:53 AM

- A *right circular cone* has been commonly referred to as a *cone* since the elementary years; we will continue to use *cone* to refer to a *right circular cone.*



Feb 27-6:53 AM

For pyramids, in addition to the base, we have lateral faces and edges.

- Name a lateral face and edge in Figure 5 and explain how you know it is a lateral face.

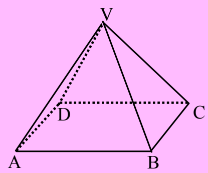
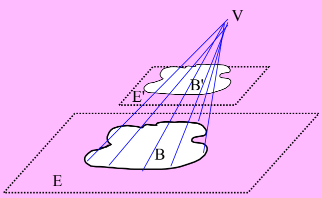


Figure 5

Feb 27-6:53 AM

Look at the general cone in Figure 6. The plane E' is parallel to E and is between the points V and the plane E. The intersection of the general cone with E' gives a cross-section B'.

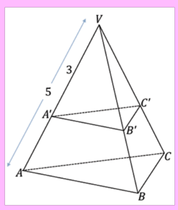


Feb 27-6:53 AM

Example 1

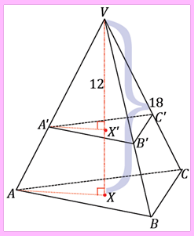
ΔABC

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section ΔA'B'C'. If the area of ΔABC is 25 mm², what is the area of ΔA'B'C'?



Feb 27-6:53 AM

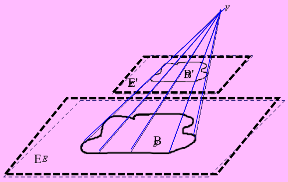
In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\Delta A'B'C'$. The altitude form V is drawn; the intersection of the altitude with the base is X, and the intersection ift the altitude withe the cross-section is X'. If the distance from X to V is 18 mm, the distance from X' to V is 12 mm, and the area of $\Delta A'B'C'$ is 28 mm^2, what is the area of ΔABC?



Feb 27-6:53 AM

- How can this result be used to show that any pyramid (i.e., those with polygonal bases rather than those with triangular bases) has cross-sections similar to the base?
  - *Whatever polygon represents the base of the pyramid, we can cut the pyramid up into a bunch of triangular regions. Then, the cross-section will be a bunch of triangles that are similar to the corresponding triangles in the base. So, the cross-section as a whole is similar to the base.*
- Observe that while we've only proven the result for pyramids, it does generalize for general cones, just as we suspected when we discussed dilations.

Feb 27-6:53 AM



Feb 27-6:53 AM

Exercise 1

The area of the base of a cone is 16 and the height is 10. Find the area of a cross-section that is distance 5 from the vertex.

Feb 27-6:53 AM

General cone cross-section theorem: If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words.

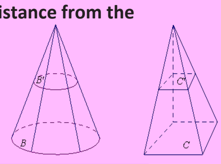
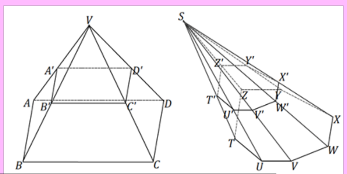


Figure 8

Feb 27-6:53 AM

The following pyramids have equal altitudes, and both bases are equal in area and are coplanar. Both pyramids' cross-section are also coplanar. If $BC = 3\sqrt{2}$ and $B'C' = 2\sqrt{3}$, and the area of TUVWXYZ is 30 $units^2$, what is the area of cross-section A'B'C'D'?



Feb 27-6:53 AM

Lesson Summary

CONE: Let B be a region in a plane E and V be a point not in E. The *cone with base B and vertex V* is the union of all segments $\overline{VP}$ for all points P in B.

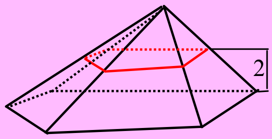
If the base is a polygonal region, then the *cone* is usually called a *pyramid*.

RECTANGULAR PYRAMID: Given a rectangular region B in a plane E and a point V not in E, the *rectangular pyramid with base B and vertex V* is the union of all segments $\overline{VP}$ for points P in B.

LATERAL EDGE AND FACE OF A PYRAMID: Suppose the base B of a pyramid with vertex V is a polygonal region and P_i is a vertex of B. The segment $\overline{P_iV}$ is called a *lateral edge* of the pyramid. If $\overline{P_iP_{i+1}}$ is a base edge of the base B (a side of B), and F is the union of all segments $\overline{PV}$ for P in $\overline{P_iP_{i+1}}$, then F is called a *lateral face* of the pyramid. It can be shown that the face of a pyramid is always a triangular region.

Feb 27-6:53 AM

1. The base of a pyramid has area 4. A cross-section that lies in a parallel plane that is distance of 2 from the base plane has an area of 1. Find the height, h, of the pyramid.



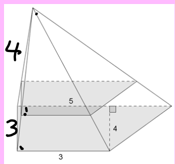
Feb 27-6:53 AM

2. The base of a pyramid is a trapezoid. The trapezoidal bases have lengths of 3 and 5, and the trapezoid's height is 4. Find the area of the parallel slice that is three-fourths of the way from the vertex to the base.

$A_{Base} = 4\left(\frac{3+5}{2}\right) = 16$

$A_{slice} = 16 \cdot \left(\frac{3}{4}\right)^2$

$9u^2$



Feb 27-6:53 AM

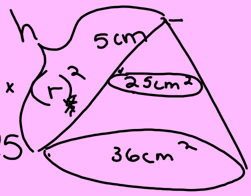
3. A cone has base area 36 cm^2. A parallel slice 5 cm from the vertex has area 25 cm^2 Find the height of the cone.

Area = $A_{original} \times (r)^2$

$\left(\frac{5}{h}\right)^2 \cdot 36 = 25$

r^2 = areas

$\frac{25}{h^2} = \frac{25}{36}$



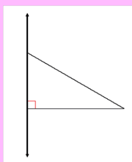
$\frac{25h^2}{25} = \frac{25 \cdot 36}{25}$

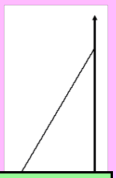
$\sqrt{h^2} = \sqrt{36}$

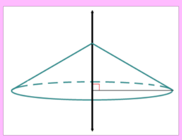
$h = 6$

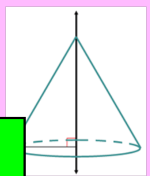
Feb 27-6:53 AM

4. Sketch the figures formed if the triangular regions are rotated around the provided axis:



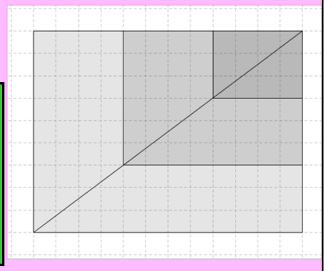






Feb 27-6:53 AM

5. Liza drew the top view of a rectangular pyramid with two cross-sections as shown in the diagram and said that her diagram represents one, and only one, rectangular pyramid. Do you agree or disagree with Liza? Explain.



Feb 27-6:53 AM

6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in^2. Find the area of the base.

[redacted]

Feb 27-6:53 AM

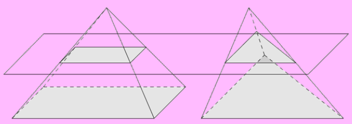
7. A general cone has base area 3 units^2. Find the area of the slice of the cone that is parallel to the base and 2/3 of the way from the vertex to the base.

$(3) \cdot \left(\frac{2}{3}\right)^2 = \frac{4}{3}$ units^2

[redacted]

Feb 27-6:53 AM

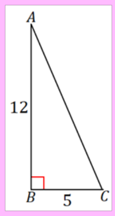
8. A rectangular cone and a triangular cone have bases with the same area. Explain why the cross-sections for the cones halfway between the base and the vertex have the same area.



[redacted]

Feb 27-6:53 AM

9. The following right triangle is rotated about side AB.
What is the resulting figure, and what are its dimensions?

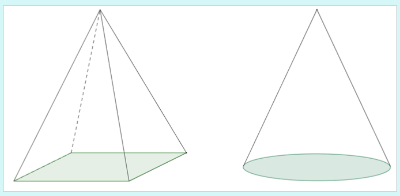


[redacted]

Feb 27-6:53 AM

Homework

The diagram below shows a circular cone and a general pyramid. The bases of the cones are equal in area, and the solids have equal heights.

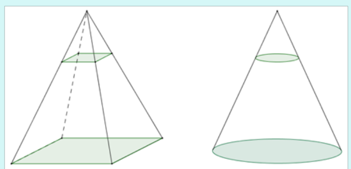


a. Sketch a slice in each cone that is parallel to the base of the cone and 2/3 closer to the vertex than the base plane.

b. If the area of the base of the circular cone is 616 units^2, find the exact area of the slice drawn in the pyramid.

Mar 2-8:24 AM

a. Sketch a slice in each cone that is parallel to the base of the cone and 2/3 closer to the vertex than the base plane.



Mar 2-8:24 AM

3. A cone has base area 36 cm^2. A parallel slice 5 cm from the vertex has area 25 cm^2 Find the height of the cone.

Feb 27-6:53 AM

6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in^2. Find the area of the base.

Feb 27-6:53 AM

6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in^2. Find the area of the base.

Feb 27-6:53 AM

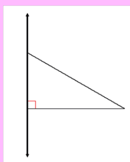
7. A general cone has base area 3 $units^2$. Find the area of the slice of the cone that is parallel to the base and 2/3 of the way from the vertex to the base.

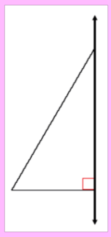
Feb 27-6:53 AM

7. A general cone has base area 3 $units^2$. Find the area of the slice of the cone that is parallel to the base and 2/3 of the way from the vertex to the base.

Feb 27-6:53 AM

4. Sketch the figures formed if the triangular regions are rotated around the provided axis:





Feb 27-6:53 AM

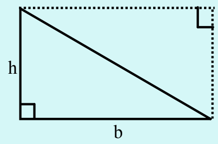
b. If the area of the base of the circular cone is 616 units^2, find the exact area of the slice drawn in the pyramid.

The distance from the slice to the vertex is 1/3 the height of the cone, so the scale factor from the base to the slice is 1/3. The areas of the planar regions are related by the square of the scale factor, of $r^2 = 1/9$.
Area (slice) =1/9(area(base))
Area(slice) = 1/9(616)
Area(slice) = 616/9 = $68.\overline{4}$

If two cones have the same base area and the same height, then cross-sections for the cones the same distance from the vertex have the same area, so the area of the slice from the pyramid is 616/9 units^2.

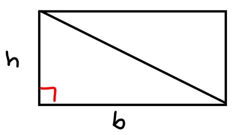
Mar 2-8:24 AM

a. Use the following image to reason why the area of a right triangle is ½bh (Area Property 2).



Mar 2-8:24 AM

opening exercise



$A_\Delta = \frac{1}{2} b \cdot h$

Oct 28-2:05 PM

We can use the formula
density = mass / volume
to find the density of a substance.

1. A square metal plate has a density of 10.2 g/cm^3 and weighs 2.193 kg.
a. Calculate the volume of the plate. (notice the units)

Mass = 2.193 kg = 2193 grams

$D = \frac{M}{V}$

$\frac{10.2}{1} = \frac{2193}{V}$

$10.2V = 2193$

$\frac{10.2V}{10.2} = \frac{2193}{10.2}$

$V = 215 cm^3$

Mar 2-8:24 AM

b. If the base of this plate has an area of 25 cm^2 determine its thickness.

$V = 215 cm^3$

$V = Bh$

$\frac{215cm^3}{25cm^2} = \frac{25cm^2 h}{25cm^2}$

$8.6 cm = h$

Mar 2-8:24 AM

2. A metal cup full of water has a mass of 1000 g. The cup itself has a mass of 214.6 g. If the cup has both a diameter and a height of 10 cm, what is the approximate density of water?

First what is the mass of the water?
The volume of the water in the cup is equal to the volume of the cylinder with the same dimensions.

Mar 2-8:24 AM

$D = \frac{m}{V}$

Mass Full Cup of H_2O − Mass Cup = Mass H_2O

1000 − 214.6 = 785.4 g = M

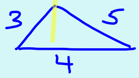
$V_{cyl} = Bh$

$= (\pi(5)^2)10 = 250\pi\ cm^3$

$D = \frac{M}{V} = \frac{785.4\ g}{250\pi\ cm^3}$ | 1 g/cm³

Nov 1-11:03 AM

3. Find the volume of a triangle with side lengths 3, 4, and 5.



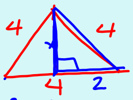
It must be a 3-Dimensional figure in order to have volume and a triangle is a 2-Dimensional figure.

Mar 2-8:24 AM

4. The base of the prism shown in the diagram consists of overlapping congruent triangles ABC, DGH. Points C, D, E, and F are midpoints of the sides of the triangles ABC and DGH. GH = AB = 4, and the height of the prism is 7. Find the volume of the prism.

equivalent

union of the two triangular prisms minus the overlap



$V = A_B \cdot h$

$V = \left(\frac{1}{2}(\sqrt{12})4\right) \cdot 7$

$x^2 + 2^2 = 4^2$

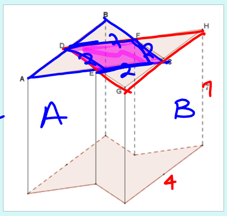
$x^2 + 4 = 16$

$x^2 = 12$

$x = \sqrt{12}$

$V_\triangle = 48.5$

$V_\square = 2 \cdot 2 \cdot 7 = 28$



$V_{A \cup B} = V_A + V_B - V_{A \cap B}$

$= 48.5 + 48.5 - 28 = 69\ units^3$

Mar 2-8:24 AM

$\overline{DC}$ connects the midpoints of $\overline{AB}$ and $\overline{GH}$ and is, therefore, the altitude of both triangles ABC and DGH. The altitude in an equilateral triangle splits the triangle into two congruent 30-60-90 triangles. Using the relationships of the legs and hypotenuse of a 30-60-90 triangle, $DC = 2\sqrt{3}$.

Volume of triangular prism with base ABC:

$$V = \frac{1}{2}(4 \cdot 2\sqrt{3}) \cdot 7$$

$$V = 28\sqrt{3}$$

The volume of the triangular prism with base DGH is also $28\sqrt{3}$ by the same reasoning.

Volume of parallelogram CEDF:

$V = 2 \cdot \sqrt{3} \cdot 7$

$V = 14\sqrt{3}$

$V(A \cup B) = V(A) + V(B) - V(A \cap B)$

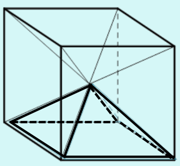
$V(\text{prism}) = 28\sqrt{3} + 28\sqrt{3} - 14\sqrt{3}$

$V(\text{prism}) = 42\sqrt{3}$

The volume of the prism is $42\sqrt{3}$.

Mar 2-8:24 AM

5. Find the volume of a right rectangular pyramid whose base is a square with side length 2 and height is 1.



Mar 2-8:24 AM

Piecing six congruent copies of the given pyramid together forms a cube with edges of length 2. The volume of the cube is equal to the area of the base times the height:

$\text{Volume}_{cube} = 2^2 \cdot 2$

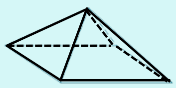
$\text{Volume}_{cube} = 8$

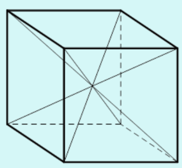
Since there are six identical copies of the pyramid forming the cube, the volume of one pyramid is equal to $\frac{1}{6}$ of the total volume of the cube:

$V_p = \frac{1}{6}(8)$

$V_p = \frac{8}{6} = \frac{4}{3}$

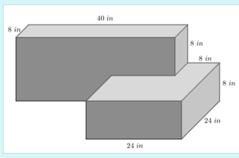
The volume of the given rectangular pyramid is $\frac{4}{3}$ cubic units.





Mar 2-8:24 AM

6. Paul is designing a mold for a concrete block to be used in a custom landscaping project. The block is shown in the diagram with its corresponding dimensions and consists of two intersection rectangular prisms. Find the volume of mixed concrete, in cubic feet, needed to make Paul's custom block.



Mar 2-8:24 AM

The volume is needed in cubic feet, so the dimensions of the block can be converted to feet:

8 in. → $\frac{2}{3}$ ft.

16 in. → $1\frac{1}{3}$ ft.

24 in. → 2 ft.

40 in. → $3\frac{1}{3}$ ft .

The two rectangular prisms that form the block do not have the same height; however, they do have the same thickness of $\frac{2}{3}$ ft.*, and their intersection is a square prism with base side lengths of* $\frac{2}{3}$ ft.*, so Volume Property 4 can be applied:*

$$V(A \cup B) = V(A) + V(B) - V(A \cap B)$$

$$V(A \cup B) = \left[\left(1\frac{1}{3} \cdot 3\frac{1}{3}\right) \cdot \frac{2}{3}\right] + \left[(2 \cdot 2) \cdot \frac{2}{3}\right] - \left[\left(\frac{2}{3} \cdot 1\frac{1}{3}\right) \cdot \frac{2}{3}\right]$$

$$V(A \cup B) = \left[\frac{80}{27}\right] + \left[\frac{8}{3}\right] - \left[\frac{16}{27}\right]$$

$$V(A \cup B) = \frac{136}{27} \approx 5.04\ ft^3$$

Paul will need just over 5 ft^3 *of mixed concrete to fill the mold.*

Mar 2-8:24 AM

Oct 21-7:17 AM

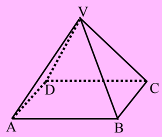


Figure 5

The triangular region AVB is defined by a side of the base, AB and vector V and is an example of a lateral face.

Feb 27-6:53 AM

Oct 31-6:34 AM