10/19Aim: General Pyramids and Cones and Their Cross-Sections

Do now: Copy the student outcome and do the opening exercise

Student Outcome:

- Students understand the definition of a *general pyramid* and *cone,* and that their respective *cross-sections* are similar to the base.
- Students show that if two cones have the same base area and the same height, then cross-sections for the cones the same distance from the vertex have the same area.

homework Exit ticket

Test Tuesday and Wednesday 3/10 and 11

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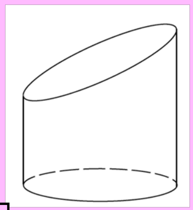
Homework answers

1. Is this a cylinder? Explain why or why not?

This is not a cylinder because the bases are not parallel to each other.



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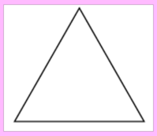
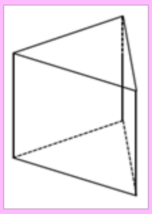
2. For each of the following cross-secti the figure from which the cross-section w





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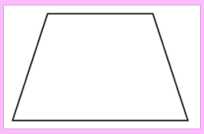
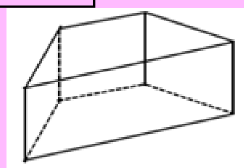
2. For each of the following cross-sections, sketch the figure from which the cross-section was taken.





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Hint for Opening exercise
General cylinders
prism
Pointy with polygonal bases
pointy with curved bases

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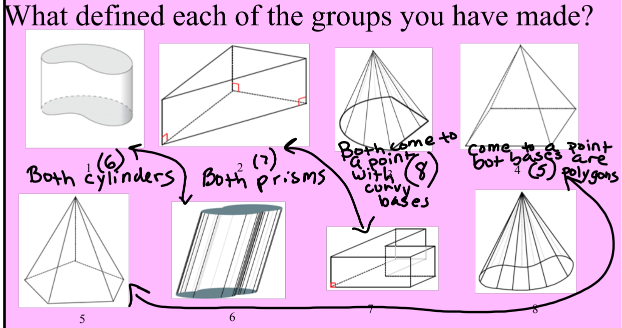
Opening exercise
Group the following images by shared properties.
What defined each of the groups you have made?



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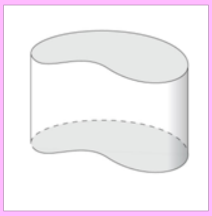
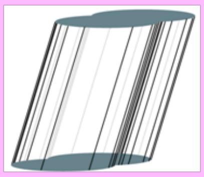
Group A: General cylinders, images 1 and 6.





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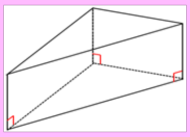
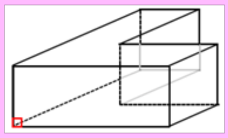
Group B: Prisms, images 2 and 7.





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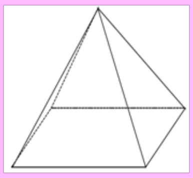
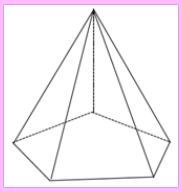
Group C: Figures that come to a point with a polygonal base, images 4 and 5.





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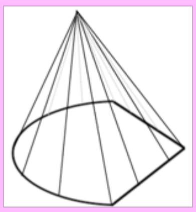
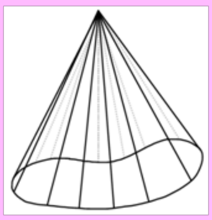
Group D: Figures that come to a point with a curved or semi-curved region as a base, images 3 and 8.





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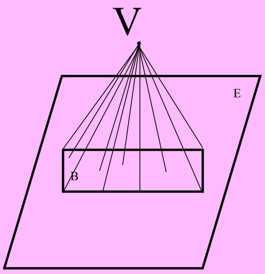
Rectangular Pyramid: Given a rectangular region B in a plane E and a point V not on E, the rectangular pyramid with base B and vertex V is the collection of all segments $\overline{VP}$ for any point in B.



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General cone: Let B be a region in a plane E and V be a point not in E. The cone with base B and vertex V is the union of all segments VP for all points P in B.

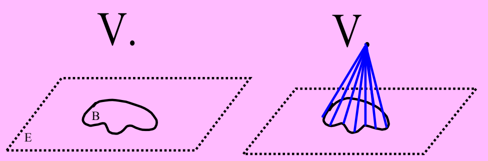

figure 1 figure 2

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What are the differences between a Rectangular Pyramid and a General Cone?

rectangular pyramid has a rectangular base. A general cone can have any region for a base.

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Important facts

- Like a general cylinder, a general cone is named by its base.
- A general cone with a disk as a base is called a *circular cone*.
- A general cone with a polygonal base is called a *pyramid*. Examples of this include a rectangular pyramid or a triangular pyramid

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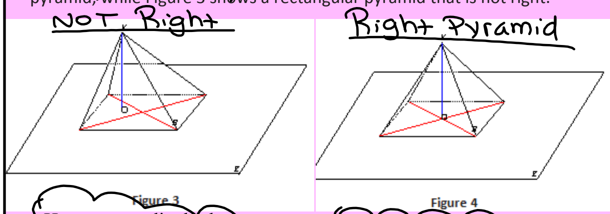
- A general cone whose vertex lies on the perpendicular line to the base and that passes through the center of the base is a *right cone* (or a *right pyramid* if the base is polygonal). Figure 4 shows a right rectangular pyramid, while Figure 3 shows a rectangular pyramid that is not right.



Has a perpendicular but not in the center.

Perpendicular and forms a right angle with the center of the base

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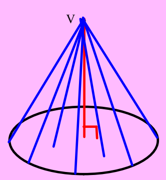
- A *right circular cone* has been commonly referred to as a *cone* since the elementary years; we will continue to use *cone* to refer to a *right circular cone.*



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For pyramids, in addition to the base, we have lateral faces and edges.

- Name a lateral face and edge in Figure 5 and explain how you know it is a lateral face.

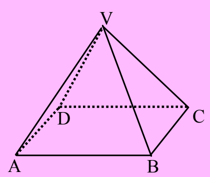

Figure 5

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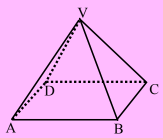

Figure 5

The triangular region AVB is defined by a side of the base, AB and vector V and is an example of a lateral face.

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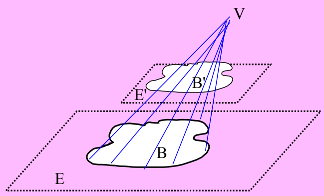
Look at the general cone in Figure 6. The plane E' is parallel to E and is between the points V and the plane E. The intersection of the general cone with E' gives a cross-section B'.



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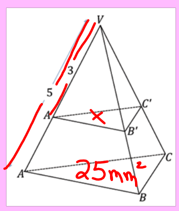
Example 1

ΔABC

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section ΔA'B'C'. If the area of ΔABC is 25 mm², what is the area of ΔA'B'C'?



scale factor: $\left(\frac{3}{5}\right)$

ratio of areas: $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$

$A_{A'B'C'} = \frac{9}{25}\left(25\right) = 9\,mm^2$

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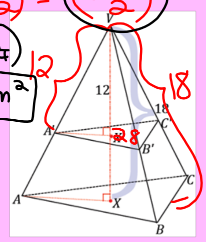
In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section ΔA'B'C'. The altitude form V is drawn; the intersection of the altitude with the base is X, and the intersection ift the altitude withe the cross-section is X'. If the distance from X to V is 18 mm, the distance from X' to V is 12 mm, and the area of ΔA'B'C' is 28 mm², what is the area of ΔABC?

$r = \left(\frac{18}{12}\right) = \left(\frac{3}{2}\right)$

Ratio of areas $= \left(\frac{3}{2}\right)^2 = \left(\frac{9}{4}\right)$

Area $= \left(\frac{9}{4}\right) \times 28 = \boxed{63\,mm^2}$



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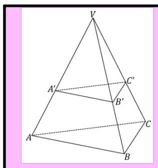

*Look at plane ABV. Can you describe a dilation of this plane that would take AB to A'B'? Remember to specify a center and a scale factor.
A dilation would have a center V and scale factor k = A'V/AV= B'V/BV'
*Do the same for plane BCV.
The scale factor for this dilation would also be k =B'/BV = C'V/CV
*What about plane CAV?
The scale factor is still k = C'V/CV = A'V/AV
*Since the corresponding sides are related by the same scale factor, what can you conclude about triangles ΔABC and ΔA'B'C'?
ΔABC ~ ΔA'B'C'by the SSS similarity criterion.

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- How can this result be used to show that any pyramid (i.e., those with polygonal bases rather than those with triangular bases) has cross-sections similar to the base?
  - *Whatever polygon represents the base of the pyramid, we can cut the pyramid up into a bunch of triangular regions. Then, the cross-section will b a bunch of triangles that are similar to the corresponding triangles in the base. So, the cross-section as a whole is similar to the base.*
- Observe that while we've only proven the result for pyramids, it does generalize for general cones, just as we suspected when we discussed dilations.

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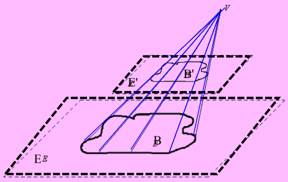

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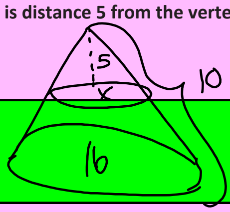
Exercise 1 B

The area of the base of a cone is 16 and the height is 10. Find the area of a cross-section that is distance 5 from the vertex.



$r = \frac{5}{10} = \frac{1}{2}$

$r = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\text{Area C.S.} = \frac{1}{4}(16) = 4u^2$

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General cone cross-section theorem: If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words.

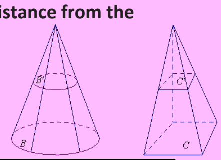

Figure 8

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*Let the bases of the cones B and C inFigure 8 be such that
(1) Area (B) =Area(C)
(2) the height of each cone is h,
(3) the distance from each vertex B' and to C' are both h'.
*How can we show that Area (B') =Area(C')?
Area (B') = $(h'/h)^2$Area(B)
Area (C') = $(h'/h)^2$Area(C)
*Since Area (B) = Area(C), thenArea (B') = Area(C')

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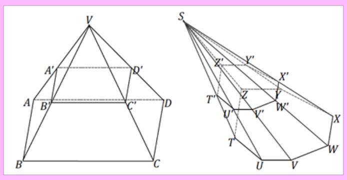
The following pyramids have equal altitudes, and both bases are equal in area and are coplanar. Both pyramids' cross-section are also coplanar. If BC = 3√2 and B'C' = 2√3, and the area of TUVWXYZ is 30 units^2, what is the area of cross-section A'B'C'D'?



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Lesson Summary

CONE: Let B be a region in a plane E and V be a point not in E. The *cone with base B and vertex V* is the union of all segments $\overline{VP}$ for all points P in B.

If the base is a polygonal region, then the *cone* is usually called a *pyramid*.

RECTANGULAR PYRAMID: Given a rectangular region B in a plane E and a point V not in E, the *rectangular pyramid with base B and vertex V* is the union of all segments $\overline{VP}$ for points P in B.

LATERAL EDGE AND FACE OF A PYRAMID: Suppose the base B of a pyramid with vertex V is a polygonal region and P_i is a vertex of B. The segment $\overline{P_iV}$ is called a *lateral edge* of the pyramid. If $\overline{P_iP_{i+1}}$ is a base edge of the base B (a side of B), and F is the union of all segments $\overline{PV}$ for P in $\overline{P_iP_{i+1}}$, then F is called a *lateral face* of the pyramid. It can be shown that the face of a pyramid is always a triangular region.

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1. The base of a pyramid has area 4. A cross-section that lies in a parallel plane that is distance of 2 from the base plane has an area of 1. Find the height, h, of the pyramid.

$\text{ratio of areas} = \sqrt{r^2} = \sqrt{\frac{1}{4}}$

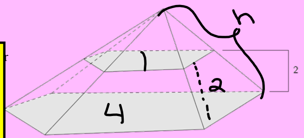
$r = \frac{1}{2} = \frac{h-2}{h}$

$h = 2(h-2)$

$h = 2h-4$

$-h = -4$

$h = 4$



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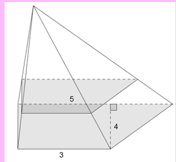
2. The base of a pyramid is a trapezoid. The trapezoidal bases have lengths of 3 and 5, and the trapezoid's height is 4. Find the area of the parallel slice that is three-fourths of the way from the vertex to the base.



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3. A cone has base area 36 cm^2. A parallel slice 5 cm from the vertex has area 25 cm^2 Find the height of the cone.

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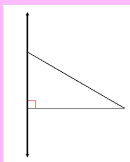
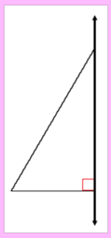
4. Sketch the figures formed if the triangular regions are rotated around the provided axis:





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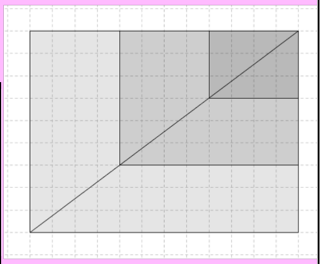
5. Liza drew the top view of a rectangular pyramid with two cross-sections as shown in the diagram and said that her diagram represents one, and only one, rectangular pyramid. Do you agree or disagree with Liza? Explain.



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6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in^2. Find the area of the base.

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7. A general cone has base area 3 units^2. Find the area of the slice of the cone that is parallel to the base and 2/3 of the way from the vertex to the base.

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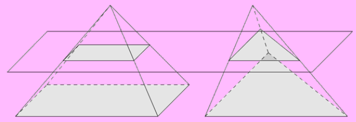
8. A rectangular cone and a triangular cone have bases with the same area. Explain why the cross-sections for the cones halfway between the base and the vertex have the same area.



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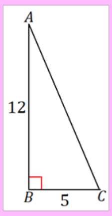
9. The following right triangle is rotated about side AB.
What is the resulting figure, and what are its dimensions?



the resulting figure is a cone
with radius 5 and height 12.

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